



- 24. If the moment generating function of a random variable is  $M_x(t) = (0.4e^t + 0.6)^3$ , find the moment generating function of  $Y = 3X + 2$ .
- 25. If  $f(x) = 6x(1-x)$ ,  $0 \leq x \leq 1$ , check whether  $f(x)$  is a pdf. Find b if  $P(X < b) = P(X > b)$ .
- 26. Let X be a random variable with the following probability distribution :  
X : -3, 6, 9 with probabilities 1/6, 1/2, 1/3 respectively, find  $E(X)$ ,  $E(X^2)$  and  $E(2X + 1)^2$ .

PART - D

Answer any 2 questions. Each question carries 6 marks. (2x6=12)

- 27. If the p.d.f. of a random variable is given by  $f(x) = \frac{1}{\theta}$ ,  $0 < x < \theta$  derive the expression for its characteristic function. Hence find its mean and variance.
- 28. Let variate X have the distribution  $P(X = 0) = P(X = 2) = p$ ;  $P(X = 1) = 1 - 2p$ , for  $0 \leq p \leq 0.5$ . For what p is the Var (X) a maximum? What is the maximum value of Variance?
- 29. A fair coin is tossed thrice, X stands for the number of heads in the first two tosses and Y stands for the number of heads in all the three tosses. Write down the joint p.d.f. of (X, Y). Hence find the marginal distributions of X and Y.
- 30. Let X and Y be two random variables having the joint distribution

X \ Y	1	2	3
1	5/27	4/27	2/27
2	1/27	3/27	3/27
3	3/27	4/27	2/27

Obtain  $E(X)$ ,  $E(Y)$ ,  $E(X+Y)$ ,  $V(X)$  and  $V(Y)$ .



Reg. No. : .....

Name : .....

**First Semester B.Sc. Honours in Mathematics (CBCSS – OBE –  
Supplementary/Improvement) Examination, November 2024  
(2021 to 2023 Admission)**

Core Course

1B03BMH : LOGIC, SETS AND PROBABILITY THEORY

Time : 3 Hours

Max. Marks : 60

PART - A

Answer any 4 questions. Each question carries 1 mark. (4x1=4)

- 1. Define the characteristic function of a continuous random variable X.
- 2. Write down the negation of the statement "S or T".
- 3. State De-Morgan's laws of probability.
- 4. If  $V(X) = V(Y) = 0.25$ , find  $V(X - Y)$ .
- 5. Give an example of a discrete random variable.

PART - B

Answer any 6 questions. Each question carries 2 marks. (6x2=12)

- 6. If  $S = \{a, b, c, d, e, f, g, h\}$ ,  $A = \{a, c, d, h\}$ ,  $B = \{b, c, e, f\}$  and  $C = \{a, c, b, d, f\}$ , establish the associative laws of events.
- 7. For a continuous random variable X, show that  $E(aX + b) = aE(X) + b$ .
- 8. Find the pdf of X if X is the sum of the faces shown when two dice are rolled.
- 9. Define the moment generating function of a discrete random variable. If  $M_x(t)$  is the moment generating function of a random variable X, write down the moment generating function of 5X?
- 10. If  $V(X) = 4$  and  $V(Y) = 2$ , find a)  $V(X + Y)$  and  $V(2X - 3Y)$ .
- 11. If F is the density function of the random variable X and if  $a < b$ , then show that  $P(a < X < b) = F(b) - F(a)$ .

P.T.O.



- 12. If  $f(x, y) = kxy$ ,  $0 < x < 1$ ,  $0 < y < 1$  is the joint p.d.f. of two random variables X and Y, Find k.
- 13. An experiment consists of three independent tosses of a fair coin. Let X = The number of heads. Write down the sample space and the event 'At most one head'.
- 14. If the distribution function of a random variable is  $F(x) = (3x^2 - x^3)/4$ ,  $0 \leq x \leq 2$ , find its probability distribution function.

PART - C

Answer any 8 questions. Each question carries 4 marks. (8x4=32)

- 15. Identify the set  $A = \{x \in \mathbb{R} : ||x - 2| - |x - 4|| = 2\}$ .
- 16. If the probability distribution of X is given by
 

X	0	1	2	3
P(X)	1/3	1/2	0	1/6

 Find  $V(X)$  and  $V(Y)$  where  $Y = 2X - 1$ .
- 17. Find the mean and variance of X if its probability density function is  $f(x) = q^x p$ ,  $x = 0, 1, 2, 3, \dots$
- 18. If  $Y = aX + b$  has mean 6 and variance 1, where X is a random variable with mean 8 and variance 16, find a and b?
- 19. Define cumulants. State the additive property of cumulants?
- 20. If  $M_x(t) = \left(1 - \frac{t}{m}\right)^{-1}$ , find the mean of X?
- 21. Define a convex function and state the Jensen's inequality.
- 22. A random variable X has the following probability distribution.
 

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	2k^2	7k^2 + k

 Find k. Also find  $P(X \geq 6)$  and  $P(0 < X < 5)$ .
- 23. If X and Y are random variables taking values  $(x, y) = (0, 0), (2, 1), (1, 1)$ , find  $E\left(X - \frac{1}{3}\right)\left(Y - \frac{2}{3}\right)$ ?