Reg. No. : ....

Name : ....

VI Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Regular/ Supplementary/Improvement) Examination, April 2023
(2016 Syllabus)
BHM 601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. If f(t) = t when t > 0, then $\mathscr{L}$[f(t)] is
2. Write the inverse Laplace transform of $\frac{1}{(s-a)^2}$.
3. Find the Fourier cosine transform of the function, $f(x)=\begin{cases}k, & \text{if } 0<x<a\\ 0, & \text{if } x>a\end{cases}$.
4. Find the first order Hankel transforms of $f(r) = e^{-ar}$.
5. Find the Z transform of f(n) = n.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the Laplace transform of $\sin^2 4t$.
7. Is the statement $\mathscr{L}(fg) = \mathscr{L}(f).\mathscr{L}(g)$ true ? Justify your answer.
8. Find $\mathscr{F}_c(e^{-x})$.
9. Find the first order Hankel transforms of $f(r) = \frac{\sin ar}{r}$.
10. Find the Mellin transform of the function $f(x) = x^m e^{-nx}$, m,n > 0.
11. Find $Z\{n^2\}$.

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12. Find the inverse Z transform of $\exp\left(\frac{1}{z}\right)$.
13. Show that $Z(na^n) = \frac{az}{(n-a)^2}$.
14. State and prove the scaling property of Mellin transformation.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Solve the initial value problem $y'' + 9y = 10e^{-t}; y(0) = 0, y'(0) = 0$.
16. Find the inverse transform of $\ln\left(\frac{s^2+\omega^2}{s^2}\right)$.
17. Solve the Voltra integral equation $y(t) - \int_0^t y(\tau)\sin(t-\tau)d\tau = t$.
18. Find the Fourier integral representation of $f(x)=\begin{cases}1, & \text{if } |x|<1\\ 0, & \text{otherwise}\end{cases}$.
19. If $f(x)=\begin{cases}x^2, & \text{if } 0<x<1\\ 0, & \text{if } x>1\end{cases}$. Find $\mathscr{F}_c(f)$.
20. Find the Fourier transform of $f(x)=\begin{cases}1, & \text{if } |x|<1\\ 0, & \text{otherwise}\end{cases}$.
21. Prove that $\mathscr{M}\left(\frac{1}{e^x+e^{-x}}\right) = \Gamma(p).L(p)$ where $L(p) = \sum_{n=0}^{\infty}(-1)^n \frac{1}{(2n+1)^p}$.
22. If $\mathscr{M}\{f(x)\} = \tilde{f}(p)$ and $\mathscr{M}\{g(x)\} = \tilde{g}(p)$, then prove that
$\mathscr{M}\{f(x)g(x)\} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\tilde{f}(s)\tilde{g}(p-s)ds$.
23. If $\tilde{f}_n(k) = \mathscr{H}\{f(n)\}$, prove that $\mathscr{H}_n\{f'(r)\} = \frac{k}{2n}\left[(n-1)\tilde{f}_{n+1}(k) - (n+1)\tilde{f}_{n+1}(k)\right]$, n ≥ 1.

24. If f(n) is a periodic sequence of integral period N, then prove that
$Z\{f(n)\} = \left(\frac{z^N}{z^N-1}\right)\sum_{k=0}^{N-1} f(k)z^{-k}$.
25. Solve the difference equation $u(n+2) - 5u(n+1) - 6u(n) = 2^n$, u(0)=1, u(1) = 0.
26. Find $\sum_{n=0}^{\infty} a^n \sin nx$ using Z transform.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Solve the initial valued problem :
$y_1' + y_2 = 0, y_1 + y_2' = 2\cos t$
$y_1(0) = 1, y_2(0) = 0$
28. Let f(x) be continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$ and let f'(x) be absolutely integrable on the x-axis. Then prove the following :
a) $\mathscr{F}_c\{f'(x)\} = \omega\,\mathscr{F}_s\{f(x)\} - \sqrt{\frac{2}{\pi}}f(0)$
b) $\mathscr{F}_s\{f'(x)\} = -\omega\,\mathscr{F}_c\{f(x)\}$.
29. If $\tilde{f}_n(k) = \mathscr{H}\{f(n)\}$, prove that
$\mathscr{H}_n\left\{\left(\nabla^2 - \frac{n^2}{r^2}\right)f(r)\right\} = \mathscr{H}_n\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) - \frac{n^2}{r^2}f(r)\right\} = -k^2\tilde{f}_n(k)$.
30. If $Z\{f(n)\} = F(z)$, then prove that $\lim_{n\to\infty} f(n) = \lim_{z\to 1}(z-1)F(z)$, provided the limits exist.

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