K23U 1182

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE-Regular) Examination, April 2023 (2021 Admission)

4B15 BMH : INTRODUCTION TO ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **one** mark. (4×1=4)

1. Let S be a set and let f, g and h be functions mapping S into S. Then prove that fo(goh) = (fog)oh.
2. Find the generators of $\mathbb{Z}_4$.
3. State Cayley's theorem.
4. Give an example of vector space.
5. What is linear operators ?

SECTION – B

Answer **any 6** questions. **Each** question carries **two** marks. (6×2=12)

6. Prove that left and right cancellation laws hold in a group G.
7. What is a group and give an example of nonabelian group ?
8. Find the orbit of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 34 & 5 & 67 & 8 \\ 3 & 8 & 67 & 4 & 15 & 2 \end{pmatrix}$ in S_8.
9. Show that every permutation σ of a finite set is a product of disjoint cycles.
10. Prove that for any x in a vector space (–1). x = – x.
11. Show that $W = \left\{ \begin{pmatrix} x \\ y \\ 0 \end{pmatrix} : x, y \in \mathbb{R} \right\}$ is a vector space.

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12. Define column space and row space.
13. Let A be an m × n matrix. Let T be a mapping given by $T(x) = Ax$, $x \in \mathbb{R}^n$. Show that T is a linear transformation.
14. Show that $B = \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

SECTION – C

Answer **any 8** questions. **Each** question carries **four** marks. (8×4=32)

15. State and prove division algorithm for $\mathbb{Z}$.
16. Prove or disprove * defined on $\mathbb{Q}^+$ by $a * b = ab/2$ is binary operation.
17. Show that a subset H of a group G is a subgroup of G if and only if,
    1) H is closed under the binary operation of G.
    2) the identity element e of G is in H.
    3) for all a ∈ H it is true that $a^{-1} \in H$ also.
18. Show that S_n is nonabelian for $n \geq 3$.
19. Find the maximum possible order for an element in S_7.
20. Let G and G′ be groups and let $\phi : G \to G'$ be a one to one function such that $\phi(xy) = \phi(x)\phi(y)$ for all x, y ∈ G. Then prove $\phi[G]$ is a subgroup of G′ and φ is an isomorphism.
21. Show that $S = \{x \mid x = tv, t \in \mathbb{R}\}$ where $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is a subspace of $\mathbb{R}^2$.
22. Prove that for any m × n matrix A . N(A) is a subspace of $\mathbb{R}^n$.
23. If $X = \{v_1, v_2, v_3, \dots v_k\}$ is a set of vectors of a vector space V. Prove that Lin (X) is the smallest subspace of V containing $v_1, v_2, \dots, v_k$.
24. Find the bases for null space and range of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$. Given by $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 + 2x_3 \\ x_1 + x_3 \\ 2x_1 + x_2 + 3x_3 \end{pmatrix}$ and verify rank-nullity theorem.

25. Find the basis of null space of the matrix $A = \begin{pmatrix} 1 & 1 & 2 & -1 \\ 2 & 0 & 1 & 1 \\ 9 & -1 & 3 & 4 \end{pmatrix}$ and verify rank nullity theorem.
26. Find the matrix representation of $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y + z \\ x - y \\ x + 2y - 3z \end{pmatrix}$.

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks. (2×6=12)

27. Prove that a subgroup of a cyclic group are cyclic.
28. Find the multiplication table for D_4.
29. State and prove Rank nullity theorem for matrix.
30. Prove that similarity is an equivalence relation.

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