**K23U 0548**

Reg. No. : ..............................

Name : ..............................

**VI Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Regular/ Supplementary/Improvement) Examination, April 2023**
**(2016 Syllabus)**
**BHM 602 : TOPOLOGY**

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of the **five** questions. **Each** question carries 1 mark : (4×1=4)

1. Give an example of a topology on the set X = {1, 2, 3, 4}.
2. Define an embedding.
3. Give an example of a nonempty set whose interior is empty.
4. Give an example of space that is Lindeloff but not compact.
5. State Tietze characterisation of normality.

SECTION – B

Answer **any six** questions out of the **nine** questions. **Each** question carries 2 marks : (6×2=12)

1. Let $(X, \tau)$ be a space and $\mathscr{B} \subset \tau$. If for any $x \in G$, where G is an open set, there exist $B \in \mathscr{B}$ such that $x \in B \subset G$, then prove that $\mathscr{B}$ is a base for $\tau$.
2. Prove that in discrete topology, the only convergent sequences are those which are eventually constant.
3. Give an example, with proper justification, of two distinct topologies on a set X which induces the same subspace topology on some $Y \subset X$.
4. Is $\bar{A} \cap \bar{B} = \overline{A \cap B}$ ? Justify your answer.

P.T.O.



5. Prove that a subset of a topological space is open if and only if it is a neighbourhood of each of its points.
6. Prove that the continuous image of a compact space is compact.
7. Let C be a connected subset of a space X. For any set D such that $C \subset D \subset \bar{C}$ prove that D is connected.
8. Prove that limits of sequences are unique in a Hausdorff space.
9. Prove that, for any x in a regular space X and an open set G containing x, there exist an open set H containing x such that $\bar{H} \subset G$.

SECTION – C

Answer **any eight** questions out of the **twelve** questions. **Each** question carries 4 marks : (8×4=32)

1. Define cofinite topology on a set X. Show that it is a topology on X.
2. Given a collection $\mathscr{S}$ of subsets of X, is there a topology $\tau$ on X having $\mathscr{S}$ as a sub-base ? Justify your answer.
3. Prove that metrisability is a hereditary property.
4. Let $(X, \tau)$, $(Y, \mathscr{U})$ be spaces and $f : X \to Y$ be function. Prove that for every $V \in \mathscr{U}$ we have $f^{-1}(V) \in \tau$ if there exists a sub-base $\mathscr{S}$ for $\mathscr{U}$ such that $f^{-1}(V) \in \tau$ for every $V \in \mathscr{S}$.
5. Prove that a subset A of a space X is dense in X if and only if for every non-empty open subset B of X we have $A \cap B \neq \phi$.
6. Prove that there is a one-to-one correspondence between the set of topologies on a set and the set of all nearness relations on that set.
7. Let $(X, \tau)$, be a space and $A \subset X$. Prove that A is a compact subset of X if and only if the subspace $(A, \tau/A)$ is compact.
8. Give an example of a property which is weakly hereditary but not hereditary. Justify your answer.
9. For $\mathbb{R}$ with usual topology, prove that if $X \subset \mathbb{R}$ is connected then X is an interval.
10. Prove that all metric spaces are $T_4$.



11. Let X be a Hausdorff space, $x \in X$ and F be a compact subset of X not containing x. Prove that there exist open subsets U, V such that $x \in U$, $F \subset V$ and $U \cap V = \phi$. Further, prove that a compact subset of a Hausdorff space is closed.
12. Prove that every regular, Lindeloff space is normal.

SECTION – D

Answer **any two** questions out of the **four** questions. **Each** question carries 6 marks : (2×6=12)

1. a) Prove that, if a sequence in a cofinite topology is convergent then there is at most one term in the sequence which repeats infinitely often.
   b) If a space X is second countable then prove that every open cover of X has a countable subcover.
2. Let X be a set and let $\theta : P(X) \to P(X)$ such that
   i) $A \in P(X) \Rightarrow A \subset \theta(A)$,
   ii) $\theta(\phi) = \theta$,
   iii) $\theta$ is idempotent and
   iv) $\theta$ commutes with finite unions. Prove that there is a unique topology $\tau$ on X such that $\theta$ coincides with the closure operator associated with $\tau$.
3. State and prove Lebesgue covering Lemma.
4. a) Prove that a space X is $T_1$ if and only if for any $x \in X$ the singleton set {x} is closed.
   b) Prove that regularity is a hereditary property.

______________