Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary/Improvement) Examination, November 2023
(2018 – 2020 Admissions)
BHM 501 : SPECIAL FUNCTIONS

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define power series in x.
2. Write the formula for Bessels function of first kind of order p.
3. Define regular singular point of the differential equation. $y'' + P(x)y' + Q(x)y = 0$.
4. Define Beta function.
5. Evaluate $\Gamma(-1/2)$.   **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Check the nature of the point $x = 0$ for the differential equation $(2x^2 + 2x)y'' + (1 + 5x)y' + y = 0$.
7. Find $J_{\frac{5}{2}}(x)$.
8. Prove that $\Gamma(p + 1) = p\Gamma(p)$.
9. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.
10. Prove that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

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11. State the orthogonality property of Legendre polynomial.
12. Prove that $\int_{-1}^{7} \frac{dx}{\sqrt[3]{x+1}}$ converges and find its value.
13. Prove that $B(u, v) = B(v, u)$.
14. Evaluate $\int_0^\infty \sqrt{y}e^{-y^2} dy$.   **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Prove that :
    a) $\Gamma(x + 1) = x\Gamma(x), x > 0$
    b) $\Gamma(n + 1) = n!$.
16. Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.
17. Verify that $\cos x = \lim_{a \to \infty} F\left(a, a, \frac{1}{2}, \frac{-x^2}{4a^2}\right)$.
18. Consider generating function of Legendre polynomial

    $P_0(x) + P_1(x)t + P_2(x)t^2 \ldots\ldots + P_n(x)t^n = \frac{1}{\sqrt{1-2xt+t^2}}$. Verify $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.
19. Prove that $2J_{p'}(x) = J_{p-1}(x) - J_{p+1}(x)$.
20. Write Rodrigues formula and find $P_3(x)$.
21. Consider the differential equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$. Show that the coefficients a_n are related by the recursion formula

    $a_{n+2} + \frac{(p-n)(p+n+1)}{(n+1)(n+2)} a_n = 0$.
22. Find the power series solution of differential equation $y'' + y = 0$.

23. Examine the convergence of the following integrals :
    i) $\int_1^5 \frac{1}{\sqrt{x^4 - 1}} dx$
    ii) $\int_0^3 \frac{1}{(3-x)\sqrt{x^2+1}} dx$
    iii) $\int_1^\infty \frac{\ln x}{x+a} dx$.
24. Evaluate each of the following integrals :
    i) $\int_0^1 x^4(1-x)^3 dx$
    ii) $\int_0^2 \frac{x^2}{\sqrt{2-x}} dx$
    iii) $\int_0^{\frac{\pi}{2}} \sin^6\theta \, d\theta$.
25. Given $\int_0^\infty \frac{x^{p-1}}{1+x} dx = \frac{\pi}{\sin p\pi}$. Show that $\Gamma(p)\Gamma(1 - p) = \frac{\pi}{\sin p\pi}$, $0 < p < 1$. Also evaluate $\int_0^\infty \frac{dy}{1+y^4}$.
26. Derive Rodrigues formula.   **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Show that $\int_0^{\frac{\pi}{2}} \sin^{2u-1}\theta \cos^{2v-1}\theta \, d\theta = \frac{\Gamma(u)\Gamma(v)}{2\Gamma(u+v)}$, $u, v > 0$.
28. Find the first three terms of the Legendre series of $f(x) = e^x$.
29. Determine the nature of the point at $x = \infty$ of the equation

    $y'' + \left(\frac{4}{x}\right)y' + \left(\frac{2}{x^2}\right)y = 0$ and find the corresponding exponents.
30. Find the general solution of differential equation

    $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ at the point $x = 0$.   **(2×6=12)**

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