K23U 2647

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Regular) Examination, November 2023
(2021 Admission)
5B20BMH : INTEGRAL TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark. **(4×1=4)**

1. Let $f(t) = 1, t \geq 0$. Find $F(s)$.
2. Define the Heaviside Function.
3. Find the fundamental period of the function $f(x) = \sin 3x$.
4. State the convolution theorem for Fourier Integrals.
5. Give an example of a two dimensional partial differential equation.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Given that $f(t) = t^2 + t + 7$. Find $F(s)$.
7. Find the Laplace transform of coshat.
8. Find the Fourier coefficient a_0 for the function $f(x) = x, -\pi < x < \pi$ with $f(x + 2\pi) = f(x)$.
9. Show that $f =$ const is periodic with any period.

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10. Show that the function $f(x) = x\sin 2x$ is even.
11. $f(x) = 1, 0 < x < \infty$ has no Fourier Cosine Transform. Give reasons.
12. What is a Fourier Cosine Integral ? Explain.
13. Solve $u_{xx} + u_x = 0$ like an ODE.
14. If u_1 and u_2 are solutions of a homogeneous linear PDE in some region R, then $c_1u_1 + c_2u_2$ with any constants c_1 and c_2 is also a solution of that PDE in the region R.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks **each**. **(8×4=32)**

15. Solve $y'' - y = t, y(0) = y'(0) = 1$.
16. Find the inverse of $\frac{1}{s(s^2 + w^2)}$.
17. State and prove the first shifting theorem.
18. Find the Fourier series of the function
$$f(x) = \begin{cases} 0, & -2 < x < -1, \\ k, & -1 < x < 1, \\ 0, & 1 < x < 2 \end{cases} \quad \text{with } p = 2L = 4$$
19. What is a "Fourier Cosine Series" ? Write the formula for corresponding Fourier coefficients.
20. Show that the complex Fourier coefficients of an even function are real.
21. Find the Fourier cosine and sine transforms of the function $f(x) = \begin{cases} k, & 0 < x < a \\ 0, & x > a \end{cases}$.
22. With the usual notations, show that $\mathscr{F}_c\{f'(x)\} = -\sqrt{\frac{2}{\pi}}f(0) + w\mathscr{F}_s\{f(x)\}$.
23. Find the Fourier transform of $f(x) = e^{-ax}$ if $x > 0$ and $f(x) = 0$ if $x < 0$; here $a > 0$.

24. Verify that $u = e^x\cos y$ is a solution of the two dimensional Laplace equation.
25. Find the normal form and solution of the PDE $u_{xx} + 9u_{yy} = 0$.
26. Find the temperature $u(x, t)$ in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin(\pi/180)$°C and the ends are kept at 0°C. How long will it take for the maximum temperature in the bar to drop to 50°C [Physical data for copper : density 8.92 gm/cm^3, specific heat 0.092 cal/(gm°C), thermal conductivity 0.95 cal/(cmsec°C))].

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks **each**. **(2×6=12)**

27. i) Find the inverse transform of $\ln\left(\frac{s^2 + w^2}{s^2}\right)$.

    ii) With the usual notations, prove that $\mathscr{L}(af(t) + bg(t)) = a\mathscr{L}(f(t)) + b\mathscr{L}(g(t))$.
28. Find the complex Fourier series of $f(x) = e^x$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$ and obtain from it the actual Fourier series.
29. i) Find the Fourier integral representation of the function
$$f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

    ii) Derive the formula for the Fourier transform of the derivatives.
30. Derive the D'Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$.
