K23U 2644

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary/Improvement) Examination, November 2023
(2018 – 2020 Admissions)
BHM 504 : DIFFERENTIAL GEOMETRY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark. **(4×1=4)**

1. Sketch the vector field $X(p) = (1,0)$ on R^2.
2. Define a level set.
3. Find the regular points of the function $f : R^2 \to R$ defined by $f(x_1, x_2) = x_1^2 + x_2^2$.
4. Define an n – surface in R^{n+1}.
5. Find the speed of the parametrized curve $\alpha(t) = (\sin t, \cos t)$.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Sketch the graph of the function $f(x_1, x_2) = x_1 - x_2$.
7. Show that graph of any function $f : R^n \to R$ is a level set for some function $F : R^{n+1} \to R$.
8. Does the Mobius strip is orientable ? Justify your answer.
9. Find $\nabla f(p)$ for $f(x_1, x_2) = x_1^2 + x_2^2$ at $p = (1,1)$.
10. Show that the gradient of f at $p \in f^{-1}(c)$ is orthogonal to all vectors tangent to $f^{-1}(c)$ at p.

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11. Show by an example that the set of all vectors tangent at a point p of a level set need not in general be a vector subspace of R_p^{n+1}.
12. With the usual notations, show that $(\dot{X + Y}) = \dot{X} + \dot{Y}$.
13. Prove that geodesics have constant speed.
14. Given that X is parallel along α. Show that X has constant length.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks **each**. **(8×4=32)**

15. Does the integral curve of a vector field cross itself ? Justify your answer.
16. Prove that $x_1 + 3x_2 - 2x_3 = 1$ is an 2-surface in R^3.
17. Does the parametrized curve $\alpha(t) = (-\cos t, t, -\sin t)$ is a geodesic in the cylinder $x_1^2 + x_3^2 = 1$ in R^3. Justify your answer.
18. Compute $\Delta_v f$ where $f(x_1, x_2) = x_1^3 - 3x_2^2 + 3x_1x_2^2$, $v = (-1, 0, -1, 0)$.
19. Let S be an n – surface in R^{n-1} and let, $\alpha : I \to S$ be a parametrized curve, and let X be a vector field tangent to S along α. Prove that $(fX)' = f'X + fX'$.
20. Find the integral curve through $(-1, 0)$ of the vector field $X(x_1, x_2) = (x_1, x_2, -x_2, x_1)$.
21. Prove that an n – sphere is connected.
22. Sketch the cylinder over the graph of $f(x) = \sin x$.
23. State and prove Lagrange Multiplier Theorem.
24. Let $a, b, c \in R$ such that $ac - b^2 > 0$. Show that the maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $ax_1^2 + 2bx_1x_2 + cx_2^2 = 1$ are of the form $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$ where λ_1, λ_2 are the eigen values of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$.
25. Sketch the surface of revolution obtained by rotating the curve $x_2^2 = x_1$, $0 < x_1 < 1$ about the x_1 – axis.
26. Compute the Weingarten map for the cylinder $x_1^2 + x_2^2 = 1$ in R^3.

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SECTION – D

Answer **any two** questions. **Each** question carries **6** marks **each**. **(2×6=12)**

27. Let $g : I \to R$ be a smooth function and let C denote the graph of g. Show that the curvature of C at the point $(t, g(t))$ is $\frac{g''(t)}{(1+(g'(t))^2)^{\frac{3}{2}}}$.
28. Prove the following : Let S be an n surface in R^{n+1} and let $p \in S$ and let $v \in S_p$. Then there exist an open interval I containing 0 and a geodesic $\alpha : I \to S$ such that
    i) $\alpha(0) = p$, $\dot{\alpha}(0) = v$
    ii) If $\beta : \tilde{I} \to S$ is any other geodesic in S with $\beta(0) = 0$, $\dot{\beta}(0) = v$, then $\tilde{I} \subset I$ and $\alpha(t) = \beta(t)$ for all $t \in \tilde{I}$.
29. Prove that the Weingarten map is self-adjoint.
30. With the usual notations, prove that the parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.