K23U 2649

Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree
(C.B.C.S.S. – OBE – Regular) Examination, November 2023
(2021 Admission)
5B22BMH : COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark. **(4×1=4)**

1. Does the point $1 - i$ is lie on the upper half complex plane ? Justify your answer.
2. What is an open set ?
3. Evaluate $\int_0^{1+i} z^2 dz$.
4. Give an example of a convergent sequence.
5. Find the zeros of the function $f(z) = 1 + z^2$.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Write the polar form of the complex number $1 + i$.
7. Show that $\cos z = \cos x \cosh y - i \sin x \sinh y$.
8. Show that $\oint_C \frac{dz}{z} = 2\pi i$ where C is the unit circle centred at the origin oriented counter clock wisely.
9. State Cauchy's integral theorem.
10. Evaluate $\oint_C x dz$ where C is the line segment joining 1 to $1 + 2i$.
11. If $z_1 + z_2 + \ldots$ converges, show that $\lim_{m \to \infty} z_m = 0$.

12. When we say a series is absolutely convergent ? Give an example.
13. Find the Laurent series of $ze^{\frac{1}{z^2}}$ with centre 0.
14. Find the residue of $f(z) = \frac{\sin z}{z^4}$ at $z = 0$.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks. **(8×4=32)**

15. Show that $\text{Re}(\sin z)$ is harmonic.
16. Show that the set of values of $\ln(i^2)$ is differ from the set of values of $2 \ln(i)$.
17. Show that $\cosh^2 z - \sinh^2 z = 1$.
18. Find an upper bound for the absolute value of the integral $\oint_C z^2 dz$, C is a straight line segment from 0 to $1 + i$.
19. Sketch and represent parametrically the curve $|z - 2 + 3i| = 4$ counter clock wisely.
20. Show that if a sequence converges, its limit is unique.
21. State and Prove the Ratio Test.
22. Show that the geometric series $\sum_{m=0}^{\infty} q^m$ converges if $|q| < 1$ and diverges otherwise.
23. Find the radius of convergence of the power series $\sum_{n=0}^{\infty}\left[1 + (-1)^n + \frac{1}{2^n}\right] z^n$.
24. Integrate $f(z) = \frac{1}{z^3 - z^4}$ clockwise around the circle $|z| = \frac{1}{2}$.
25. Define the following terms with suitable examples.
    i) Pole of order m,
    ii) Essential singularity.
26. State the Residue theorem.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks. **(2×6=12)**

27. i) Find and sketch the set of points given by $|z + 1| = |z - 1|$.
    ii) Find the derivative of $f(z) = (z^2 + i)^3$.
28. State Cauchy's integral formula and using Cauchy's integral formula evaluate counter clockwisely $\oint_C \frac{dz}{z^2 - 1}$ where $C : |z + 1| = 1$.
29. i) Find the Maclaurian's series for $f(z) = \frac{1}{z^2 + 1}$.
    ii) Find the Taylor series for $f(z) = \frac{1}{z}$ about $z = 1$.
30. Evaluate the following integral, where C is the ellipse $9x^2 + y^2 = 9$, counter clockwise $\oint_C \left[\frac{ze^{\pi z}}{z^2 - 16} + ze^{\pi/z}\right] dz$.
