Reg. No. : ....

Name : ....

V Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Regular) Examination, November 2023
(2021 Admission)
5B19 BMH : ADVANCED ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find all cosets of the subgroup $4\mathbb{Z}$ of $\mathbb{Z}$.
2. Define kernel of a homomorphism.
3. State fundamental theorem of homomorphism.
4. Find the characteristic of the ring $2\mathbb{Z}$.
5. How many polynomials are there of degree ≤ 3 in $\mathbb{Z}_2[x]$?

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Prove that every group of prime order is cyclic.
7. Let $\sigma = (1, 2, 5, 4)\ (2, 3)$ in S_5. Find the index of $\langle\sigma\rangle$ in S_5.
8. Find the order of the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_4)/\langle(2, 1)\rangle$.
9. Let H be a normal subgroup of G. Prove that the cosets of H form a group G/H under the binary operation (aH) (bH) = (ab)H.
10. Prove that a factor group of a cyclic group is cyclic.
11. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.

P.T.O.

12. Compute the remainder of 8^{103} when divided by 13.
13. Let $f(x) = 2x^2 + 3x + 4$ and $g(x) = 3x^2 + 2x + 3$ in $\mathbb{Z}_6[x]$. Find f(x) + g(x) and f(x)g(x).
14. Prove that $25x^5 - 9x^4 - 3x^2 - 12$ is irreducible over Q.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Let H be a subgroup of G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Prove that $\sim_L$ is an equivalence relation on G.
16. State and prove Lagrange theorem.
17. Let γ be the natural map of $\mathbb{Z}$ into $\mathbb{Z}_n$ given by $\gamma(m) = r$, where r is the remainder obtained by the division algorithm when m is divided by n. Prove that γ is a homomorphism.
18. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.
19. Give an example to show that the converse of Lagrange theorem does not hold.
20. Let $\phi : G \to G'$ be a group homomorphism and let N be a normal subgroup of G. Show that $\phi[N]$ is a normal subgroup of $\phi[G]$.
21. Prove that every finite integral domain is a field.
22. State and prove Euler's theorem.
23. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$.
24. Prove that the set R[x] of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication.
25. State and prove factor theorem.
26. State and prove Eisenstein criterion.

K23U 2646 -2-

12. Compute the remainder of 8^{103} when divided by 13.
13. Let $f(x) = 2x^2 + 3x + 4$ and $g(x) = 3x^2 + 2x + 3$ in $\mathbb{Z}_6[x]$. Find f(x) + g(x) and f(x)g(x).
14. Prove that $25x^5 - 9x^4 - 3x^2 - 12$ is irreducible over Q.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Let H be a subgroup of G. Let the relation $\sim_L$ be defined on G by $a \sim_L b$ if and only if $a^{-1}b \in H$. Prove that $\sim_L$ is an equivalence relation on G.
16. State and prove Lagrange theorem.
17. Let γ be the natural map of $\mathbb{Z}$ into $\mathbb{Z}_n$ given by $\gamma(m) = r$, where r is the remainder obtained by the division algorithm when m is divided by n. Prove that γ is a homomorphism.
18. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.
19. Give an example to show that the converse of Lagrange theorem does not hold.
20. Let $\phi : G \to G'$ be a group homomorphism and let N be a normal subgroup of G. Show that $\phi[N]$ is a normal subgroup of $\phi[G]$.
21. Prove that every finite integral domain is a field.
22. State and prove Euler's theorem.
23. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$.
24. Prove that the set R[x] of all polynomials in an indeterminate x with coefficients in a ring R is a ring under polynomial addition and multiplication.
25. State and prove factor theorem.
26. State and prove Eisenstein criterion.