Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE-Regular) Examination, April 2023

(2021 Admission)

4B14 BMH : ADVANCED REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Find the absolute maximum and absolute minimum of $f(x) = x^2$ on the interval [0, 2].

2. Prove that every constant function on [a, b] is in R [a, b].

3. Define norm of a partition and find norm of the partition P = (0, 1, 2, 4).

4. Show that $\lim\left(\frac{x}{n}\right)^{*} = 0$.

5. Find the radius of convergence of $\sum_{n=0}^{\infty} x^n$.

SECTION – B

Answer **any 6** questions. **Each** question carries **two** marks. **(6×2=12)**

6. Define step function with example.

7. Prove that if I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Then the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

8. State fundamental theorem (first form and second form).

9. Prove that if $f \in R[a, b]$ with $f([a, b]) \subseteq [c, d]$ and let $\phi : [c, d] \to \mathbb{R}$ be continuous. Then the composition $\phi \circ f \in R[a, b]$.

10. Prove that if $f \in R[a, b]$ and if α, β, γ are any numbers in [a, b] then $\int_\alpha^\beta f = \int_\alpha^\gamma f + \int_\gamma^\beta f$.

11. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Then prove that f is continuous on A.

12. Prove that A sequence (f_n) of bounded functions on $A \subseteq \mathbb{R}$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

13. Show that $\int_1^\infty \frac{1}{x} dx$ diverges.

14. Show that $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ converges.

SECTION – C

Answer **any 8** questions. **Each** question carries **four** marks. **(8×4=32)**

15. Prove that if I be an interval and let $f : I \to \mathbb{R}$ be a continuous on I. Then f(I) is an interval.

16. Let I = [a, b] be a closed bounded interval and let $f : I \to \mathbb{R}$ be a continuous on I. Then prove that f is bounded on I.

17. Prove if $f : A \to \mathbb{R}$ is Lipchitz function then f is uniformly continuous on A. What about the converse ?

18. State and prove squeeze theorem.

19. Prove that if $f \in R[a, b]$ then the value of the integral uniquely determined.

20. If $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then prove $f \in R[a, b]$.

21. Show that sequence $F_n(x) = x^n(1 - x)$ is converges uniformly on A = [0, 1].

22. State and prove differentiation theorem.

23. Show that (x^n) converges pointwise but not uniformly convergent.

24. What about the uniform convergence of $\sum\left(\frac{1}{x^2+n^2}\right)$.

25. Let f be a nonincreasing function on [1, ∞) such that $f(x) \geq 0$. Then prove that $\sum_{n=1}^{\infty} f(n)$ will converges if $\int_1^\infty f(x)\,dx$ converges and $\sum_{n=1}^{\infty} f(n)$ diverges if $\int_1^\infty f(x)\,dx$ diverges.

26. Show that $\beta(m, n) = \frac{\Gamma m . \Gamma n}{\Gamma(m+n)}$.

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks. **(2×6=12)**

27. State and prove maximum-Minimum Theorem.

28. State and Prove Cauchy Criterion for Riemann integrable functions.

29. Let (f_n) be a sequence of functions in R [a, b] and suppose that (f_n) converges uniformly on [a, b] to f. Then prove that $f \in R[a, b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$.

30. Show that $\int_\pi^\infty \frac{\sin x}{x} dx$ is conditionally convergent.

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