K23U 1187

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree
(CBCSS – Supplementary/Improvement) Examination, April 2023
(2017 – 2020 Admissions)
BHM 402 : ADVANCED ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Let $\Phi : G \to G'$ be a homomorphism of groups. Define $Ker(\Phi)$.
2. Define inner automorphism of a group G.
3. Is the map $\Phi : \mathbb{Z} \to \mathbb{Z}$, with $\Phi(x) = 2x$ for $x \in \mathbb{Z}$ is a ring homomorphism ? Justify.
4. Find the characteristic of the ring $\mathbb{Z}_3 \times \mathbb{Z}_4$.
5. Find the product of the polynomials $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $\mathbb{Z}_8[x]$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. State and prove Theorem of Lagrange.
7. Find all left cosets of the subgroup $3\mathbb{Z}$ of $\mathbb{Z}$.
8. Prove that a factor group of a cyclic group is cyclic.
9. Compute the factor group $(\mathbb{Z}_4 \times \mathbb{Z}_6)/\langle(0, 2)\rangle$.
10. Let G be a group. Define Z(G) and prove that Z(G) is a normal subgroup of G.
11. For integers r and s with gcd (r, s) = 1, prove that the rings $\mathbb{Z}_{rs}$ and $\mathbb{Z}_r \times \mathbb{Z}_s$ are isomorphic.

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12. Show that $2^{11213} - 1$ is not divisible by 11.
13. Let $\Phi_3 : \mathbb{Z}_7[x] \to \mathbb{Z}_7$ be the evaluation homomorphism. Compute $\Phi_3[(x^4 + 2x)(x^3 - 3x^2 + 3)]$.
14. Verify whether $f(x) = x^3 + 3x + 2$ is irreducible over $\mathbb{Z}_5$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Let γ be the natural map of $\mathbb{Z}$ into $\mathbb{Z}_n$ given by $\gamma(m) = r$, where r is the remainder given by the division algorithm when m is divided by n. Show that γ is a homomorphism.
16. Let $\Phi : G \to G'$ be a group homomorphism and let $H = Ker(\Phi)$. Then prove that $\{x \in G | \Phi(x) = \Phi(a)\}$ is the left coset aH of H.
17. Let H be a subgroup of G. Prove that the relation $\sim$ defined by $a \sim b$ if and only if $a^{-1}b \in H$ is an equivalence relation on G.
18. Prove that M is a maximal normal subgroup of G if and only if G/M is simple.
19. Prove the falsity of the converse of the theorem of Lagrange.
20. Let G be a group. Prove that the set of all commutators $aba^{-1}b^{-1}$ for $a, b \in G$ generates a subgroup C (the commutator subgroup) of G.
21. Solve the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.
22. Prove that every field is an integral domain.
23. State and prove Euler's theorem.
24. Prove that a nonzero polynomial $f(x) \in F[x]$ of degree n can have atmost n zeros in a field F.
25. Prove that $\sqrt{2}$ is not a rational number.
26. Let $f(x) \in F[x]$ and let f(x) be of degree 2 or 3. Prove that f(x) is reducible over F if and only if it has a zero in F.

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SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Let $\Phi : G \to G'$ be a group homomorphism. If H is a subgroup of G then prove that $\Phi[H]$ is a subgroup of G'.
28. Let H be a subgroup of G. Prove that H is normal if and only if $(aH)(bH) = (ab)H$, $\forall a, b \in G$.
29. Let R be a ring with additive identity 0. Then for $a, b \in R$, prove that
    a) $0a = a0 = 0$,
    b) $a(-b) = (-a)b = -(ab)$,
    c) $(-a)(-b) = ab$.
30. State and prove the division algorithm for F[x].