Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE-Regular) Examination, April 2023
(2021 Admission)

4B16 BMH : INTRODUCTION TO PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Write one dimensional wave equation.
2. Write the Charpit's auxiliary equations of the partial differential equation $f(x, y, z, p, q) = 0$.
3. Give an example for a second order partial differential equation.
4. Solve $(D^2 - 3DD' + 6D'^2)z = 0$.
5. Write the general solution of $(bD - aD' - c)z = 0$. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Form a partial differential equation by eliminating arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2$.
7. Find the general solution of $x\frac{\partial z}{\partial x} + 3z = x^2$.
8. Find the complete integral and singular integral of $z = px + qy - 2\sqrt{pq}$.
9. Solve $p^2 - q^2 = \lambda$.
10. Solve $xy\frac{\partial^2 z}{\partial x \partial y} = 1$.
11. Classify the PDE $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y - 2 = 0$.

P.T.O.

12. Solve $(D^4 - D'^4)z = 0$.
13. Find the particular integral of $r + s - 2t = e^{2x+y}$.
14. Find the particular integral of $(D^2 - 2DD' + D'^2)z = \tan(x + y)$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Eliminate the arbitrary functions f and g from $z = x\,g(y) + y\,f(x)$ and form the partial differential equation.
16. Solve $xy\,dx + y^2\,dy = zxy - 2x^2$.
17. Explain the geometrical interpretation of Lagrange's linear equation.
18. Solve $p(1 + q^2) = q(z - c)$.
19. Find the complete integral of $p^2y(1 + x^2) = qx^2$.
20. Using Charpit's method, find a complete integral of the partial differential equation $px + q^2y = z$.
21. Find the surface satisfying the partial differential equation $xr + 2p = 0$ and passing through the parabolas $z = 0$, $y^2 = 4ax$ and $z = 1$, $y^2 = -4ax$.
22. Solve the partial differential equation $r = a^2t$ using Monge's method.
23. Find the region in the XY-plane in which the PDE. $[(x - y)^2 - 1]\,u_{xx} + 2u_{xy} + [(x - y)^2 - 1]u_{yy} = 0$ is hyperbolic.

24. Solve $\frac{\partial^2 z}{\partial x^2} = a^2 z$, given that $\frac{\partial z}{\partial x} = a\sin y$ and $\frac{\partial z}{\partial y} = 0$ when $x = 0$.
25. Solve $r - 2s + t = \sin(2x + 3y)$.
26. Solve $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find the family of surface orthogonal to $\phi\left[(1+x)^2 - (1+y)^2, \frac{1}{z}(2 + x + y)\right] = 0$.
28. Apply Jacobi's method to find the complete integral of $(x_2 + x_3)(p_2 + p_3)^2 + zp_1 = 0$.
29. Solve $qr + (p + x)s + yt + y(rt - s^2) + q = 0$.
30. Solve $(D^2 - D'^2 + D + 3D' - 2)z = e^{x-y} - x^2y$. **(2×6=12)**