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K23U 1188

Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – Supplementary/Improvement) Examination, April 2023
(2017 – 2020 Admissions)
BHM 403 : COMPLEX ANALYSIS, FOURIER SERIES AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Prove that $|e^z| = e^{Re\, z}$.
2. Give an example of entire function.
3. Define harmonic function.
4. Define periodic function.
5. Write one-dimensional wave equation.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the domain of the function $f(z) = \frac{z}{z+\bar{z}}$.
7. Show that the existence of the derivative of a function at a point implies the continuity of the function at that point.
8. Find the value of z for which sinh z = 0.

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9. Show that $Log(-ei) = 1 - \frac{\pi}{2}i$.
10. Solve for u given that $u_{yy} = 0$.
11. What is a linear PDE ?
12. If f(x) has a period p then prove that the period of f(ax), $a \neq 0$ is $\frac{p}{a}$.
13. Graph $f(x) = \pi - |x|$ for $-\pi < x < \pi$.
14. Prove that if $f(z) = \frac{i\bar{z}}{2}$ in the open disk $|z| < 1$, then $\lim_{z \to 1} f(z) = \frac{i}{2}$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Let $f(z) = \frac{z}{\bar{z}}$. Does $\lim_{z \to 0} f(z)$ exist ? Justify your answer.
16. If f(z) is analytic in domain D and the modulus |f(z)| is constant in D, then prove that f(z) must be a constant in D.
17. If a function f(z) = u(x, y) + iv(x, y) is analytic in a domain D, then prove that its component functions u and v are harmonic in D.
18. Find the harmonic conjugate of the function $u(x, y) = y^3 - 3x^2y$.
19. Check whether the function f(z) = (3x + y) + i(3y – x) is entire or not.
20. Find the image of the infinite strip $0 \leq y \leq \pi$, under the transformation $w = e^z$.
21. Find the Fouries series of $f(x) = \begin{cases} 2 & \text{if } -2 < x < 0 \\ 0 & \text{if } 0 < x < 2 \end{cases}$.
22. Find the Fouries series of $f(x) = \pi - |x|$ $(-\pi < x < \pi)$.
23. Is $u = \frac{1}{\sqrt{x^2 + y^2}}$ a solution of two dimensional Laplace equation ? Justify your answer.

24. Find the type and transform into normal form $u_{xx} - 2u_{xy} + u_{yy} = 0$.
25. Solve the system of PDE s $u_{xx} = 0$, $u_{yy} = 0$.
26. Find all the roots of the equation sinz = cosh4.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find the cube roots of the number – 8i.
28. State and prove the sufficient condition for differentiability of a function at a point.
29. Derive the d'Alembert's solution of the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.
30. Find the Fourier cosine series of the function $f(x) = x$ $0 < x < \frac{1}{2}$.

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