Reg. No. : ....

Name : ....

IV Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE-Regular) Examination, April 2023
(2021 Admission)
4B18 BMH : OPERATIONS RESEARCH

Time : 3 Hours Max. Marks : 60

Answer **any 4** out of 5 questions. **Each** question carries **1** mark.

1. Define convex set.
2. Define feasible solution of a general linear programming problem.
3. Define triangular basis in transportation problem.
4. Define idle time on a machine.
5. Define optimum strategy. **(4×1=4)**

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Show that the set $S=\{(x_1,x_2):3x_1^2+2x_2^2\le 6\}$ is convex.
7. Reduce the following linear programming problem to its standard form
Maximize $z^* = 3x_1 + 4x_2 + 6x_3$ subject to the constraints $2x_1 + x_2 + 2x_3 \ge 6$
$3x_1 + 2x_2 = 8$
$7x_1 - 3x_2 + 5x_3 \ge 9$, $x_1 \ge 0$, $x_2 \ge 0$ and x_3 unrestricted in sign.
8. Let f(x) be a convex function on a convex set *S*. Then prove that the set of points in *S* at which f(x) takes on its global minimum is a convex set.
9. Obtain an initial basic feasible solution to the following transportation problem using the North-West corner rule.

	D	E	F	G	Available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
Requirement	200	225	275	250	

10. Write a short note on transportation problem.

P.T.O.

11. Given below is an assignment problem, write it as a transportation problem.

	A_1	A_2	A_3
R_1	1	2	3
R_2	4	5	1
R_3	2	1	4

12. State and prove reduction theorem.
13. For the game with the following payoff matrix, determine the optimum strategies and the value of the game.

P_1

$$P_2\begin{bmatrix}5 & 1\\ 3 & 4\end{bmatrix}$$

14. Explain the rule for determining saddle point of a payoff matrix. **(6×2=12)**

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Write the dual of the linear programming problem
Minimize $z = 4x_1 + 6x_2 + 18x_3$ subject to the constraints $x_1 + 3x_2 \ge 3$
$x_2 + 2x_3 \ge 5$ and $x_1, x_2, x_3 \ge 0$.
16. Show that the following system of linear equations has a degenerate solution ; $2x_1 + x_2 - x_3 = 2$, $3x_1 + 2x_2 + x_3 = 3$.
17. Let f(x) be differentiable in its domain. If f(x) is defined on an open convex set *S*, then prove that f(x) is convex if and only if
$f(x_2) - f(x_1) \ge (x_2 - x_1)^T \nabla f(x_1)$, for all $x_1, x_2 \in S$.
18. Prove that the number of basic (decision) variables of the general transportation problem at any stage of feasible solution must be $m + n - 1$.
19. Find the initial basic feasible solution to the following transportation problem using Vogel's approximation method.

	D_1	D_2	D_3	D_4	Supply
S_1	20	25	28	31	200
S_2	32	28	32	41	180
S_3	18	35	24	32	110
Demand	150	40	180	170	

20. Write a short note on degeneracy in transportation problem.
21. A department head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks differ in their intrinsic difficulty. His estimate, of the time, is given in the matrix below :

Men	E	F	G	H
Tasks A	18	26	17	11
B	13	28	14	26
C	38	19	18	15
D	19	26	24	10

How should the tasks be allocated, one to a maximum so as to minimize the total man-hours ?

22. Describe optimal sequence algorithm.
23. Explain Hungarian assignment method.
24. Solve the following 2 × 2 graphically :

Player A \ Player B	B_1^*	B_2	B_3	B_4
A_1	2	1	0	–2
A_2	1	0	3	2

25. Solve the following game :

Player A \ Player B	I	II	III	IV
I	3	2	4	0
II	3	4	2	4
III	4	2	4	0
IV	0	4	0	8

26. Let (a_{ij}) be the m × n payoff matrix for a two person zero sum game. Then prove that $\min_{1\le j\le n}[\max_{1\le i\le m}\{a_{ij}\}] \ge \max_{1\le i< m}[\min_{1\le j\le n}\{a_{ij}\}]$. **(8×4=32)**

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Prove that a basic feasible solution to a linear programming problem must correspond to an extreme point of the set of all feasible solution and conversely.
28. Prove that all the basis for a transportation problem are triangular.
29. Describe optimum sequence algorithm.
30. Consider the two person zero sum game with the following 3 × 2 payoff matrix for player A.

Player A \ Player B	B_1	B_2
A_1	9	2
A_2	8	6
A_3	6	4

Prove that the maxmin-minimax principle. **(2×6=12)**

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