Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (CBCSS – OBE – Regular/Supplementary/Improvement) Examination, November 2023 (2021 and 2022 Admissions)
3B11 BMH : GRAPH THEORY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define simple graph.
2. Define forest.
3. Is K_4 an Euler graph ? Justify your answer.
4. Find the number of edges in complete bipartite graph $K_{m,n}$.
5. Define k-critical graph.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. State First theorem of Graph theory.
7. Draw Petersen graph and is that regular ? Justify your answer.
8. Define vertex connectivity of a graph and find the vertex connectivity of K_4.
9. Define unicyclic graphs and give an example.
10. Differentiate between Hamiltonian path and Hamiltonian cycle.
11. Define matching and give an example.

P.T.O.

12. Prove that a connected graph G with n vertices has at least n – 1 edges.
13. Is K_5 planar ? Justify your answer.
14. Let G be a graph with $\chi(G) = k$. Then prove that G has atleast k vertices such that $d(v) \geq k - 1$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries 4 marks. **(8×4=32)**

15. Prove that it is impossible to have a group of nine people at a party such that each one knows exactly five of the others in the group.
16. Define adjacency matrix. Write the adjacency matrix of K_3.
17. Define graph isomorphism and give an example.
18. If T is a tree with n vertices then prove that it has precisely n – 1 edges.
19. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.
20. Let v be a vertex of the connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v such that v is on every u-w path in G.
21. Let G be a connected graph, then prove that G is a tree iff every edge of G is a bridge.
22. Let G be a k-regular bipartite graph with k > 0. Then prove that G has a perfect matching.
23. State The Chinese Postman Problem and The Travelling Salesman Problem.
24. Let G be a simple 3-connected graph with at least five vertices. Then prove that G has a contractible edge.
25. Let G be a simple planar graph with n vertices and e edges, where $3 \leq n$. Then prove that $e \leq 3n - 6$.
26. Let G be a graph with n vertices and q edges and let w(G) denote the number of connected components of G. Then prove that $q \geq n - w(G)$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Define self-complementary graph and prove that if G is a self-complementary graph with n vertices then either n is 4t or 4t + 1 for some integer t.
28. Prove that "A graph G is connected iff it has a spanning tree".
29. If G is a simple graph with n vertices, where $n \geq 3$, and the degree $d(v) \geq \frac{n}{2}$ for every vertex v of G. Then prove that G is Hamiltonian.
30. State and prove Euler's formula.