

Reg. No. :

Name :

**III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. - O.B.E. -
Regular/Supplementary/Improvement) Examination, November 2023
(2021 and 2022 Admissions)
3B10 BMH : CALCULUS - III**

Max. Marks : 60

Time : 3 Hours

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4×1=4)

1. Write the relation between the polar coordinates (r, θ) and the rectangular coordinates of a point (x, y) .
2. Identify the surface whose equation in cylindrical coordinates is $z = r$.
3. Define vector field on \mathbb{R}^2 .
4. If $F(x, y, z) = xzi + xyzj - y^2k$, find $\text{div } F$.
5. Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.

SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 marks. (6×2=12)

6. If $R = \left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]$ then evaluate $\iint_R \sin x \cos y \, dA$.
7. Find the mass of a triangular lamina with vertices $(0, 0)$, $(1, 0)$ and $(0, 2)$ if the density function is $\rho(x, y) = 1 + 3x + y$.
8. Find the moment of inertia I_0 of a homogeneous disk D with density $\rho(x, y) = \rho$ center the origin, and radius a .
9. Find spherical coordinates of the point whose rectangular coordinates are $(0, 2\sqrt{3}, -2)$.

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10. Find the Jacobian of the transformation $x = u^2 - v^2$, $y = 2uv$.
11. Find the work done by the force field $F(x, y) = x^2i - xyj$ in moving a particle along the quarter-circle $r(t) = \cos t i + \sin t j$, $0 \leq t \leq \frac{\pi}{2}$.
12. Determine whether the vector field $F(x, y) = (x - y)i + (x - 2y)j$ is conservative.
13. Find the flux of the vector field $F(x, y, z) = zi + yj + xk$ over the unit sphere $x^2 + y^2 + z^2 = 1$.
14. Find two vector equation for the surface $z = 2\sqrt{x^2 + y^2}$ that is, the top half of the cone $z^2 = 4x^2 + 4y^2$.

SECTION - C

Answer any 8 questions out of 12 questions. Each question carries 4 marks. (8×4=32)

15. Use a double integral to find the area enclosed by one loop of the four leaved rose $r = \cos 2\theta$.
16. Evaluate $\iint_D (x + 2y) \, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
17. Find the volume of the solid bounded by the plane $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$.
18. A solid E lies within the cylinder $x^2 + y^2 = 1$ below the plane $z = 4$ and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E .
19. Derive the formula for triple integration in spherical coordinates.
20. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
21. If $F(x, y, z) = xzi + xyzj - y^2k$, find $\text{curl } F$.
22. Evaluate $\int_C (2 + x^2y) \, ds$ where C is the upper half of the unit circle $x^2 + y^2 = 1$.
23. If $F(x, y, z) = y^2i + (2xy + e^{3z})j + 3ye^{3z}k$ find a function f such that $\nabla f = F$.
24. Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$.

25. Compute the surface integral $\iint_S x^2 \, dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.
26. The temperature u in a metal ball is proportional to the square of the distance from the center of the ball. Find the rate of heat flow across a sphere S of radius a with center at the center of the ball.

SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks. (2×6=12)

27. Find the volume of the tetrahedron bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$.
28. Evaluate the integral $\iint_R e^{\frac{x+y}{x-y}} \, dA$ where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, -2)$ and $(0, -1)$.
29. Using Green's theorem evaluate $\int_C (x^4 dx + xy dy)$ where C is the triangular curve consisting of the line segments from $(0, 0)$ to $(1, 0)$, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(0, 0)$.
30. Use Stokes' Theorem to compute the integral $\iint_S \text{curl } F \cdot dS$ where $F(x, y, z) = xzi + yzj + xyk$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy -plane.