K23U 3602

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2023
(2021 and 2022 Admissions)
3B10 BMH : CALCULUS – III

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Write the relation between the polar coordinates (r, θ) and the rectangular coordinates of a point (x, y).
2. Identify the surface whose equation in cylindrical coordinates is z = r.
3. Define vector field on $\mathbb{R}^2$.
4. If $F(x, y, z) = xzi + xyzj - y^2k$, find div F.
5. Find a parametric representation of the sphere $x^2 + y^2 + z^2 = a^2$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. If $R = \left[0, \frac{\pi}{2}\right] \times \left[0, \frac{\pi}{2}\right]$ then evaluate $\iint_R \sin x \cos y \, dA$.
7. Find the mass of a triangular lamina with vertices (0, 0), (1, 0) and (0, 2) if the density function is $\rho(x, y) = 1 + 3x + y$.
8. Find the moment of inertia I_0 of a homogeneous disk D with density $\rho(x, y) = \rho$ center the origin, and radius a.
9. Find spherical coordinates of the point whose rectangular coordinates are $(0, 2\sqrt{3}, -2)$.

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10. Find the Jacobian of the transformation $x = u^2 - v^2$, $y = 2uv$.
11. Find the work done by the force field $F(x, y) = x^2 i - xyj$ in moving a particle along the quarter-circle $r(t) = \cos t\, i + \sin t\, j$, $0 \le t \le \frac{\pi}{2}$.
12. Determine whether the vector field $F(x, y) = (x - y)i + (x - 2)j$ is conservative.
13. Find the flux of the vector field $F(x, y, z) = zi + yj + xk$ over the unit sphere $x^2 + y^2 + z^2 = 1$.
14. Find two vector equation for the surface $z = 2\sqrt{x^2 + y^2}$ that is, the top half of the cone $z^2 = 4x^2 + 4y^2$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Use a double integral to find the area enclosed by one loop of the four leaved rose $r = \cos 2\theta$.
16. Evaluate $\iint_D (x + 2y)\, dA$ where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$.
17. Find the volume of the solid bounded by the plane z = 0 and the paraboloid $z = 1 - x^2 - y^2$.
18. A solid E lies within the cylinder $x^2 + y^2 = 1$ below the plane z = 4 and above the paraboloid $z = 1 - x^2 - y^2$. The density at any point is proportional to its distance from the axis of the cylinder. Find the mass of E.
19. Derive the formula for triple integration in spherical coordinates.
20. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
21. If $F(x, y, z) = xzi + xyzj - y^2k$, find curl F.
22. Evaluate $\int_C (2 + x^2 y)\, ds$ where C is the upper half of the unit circle $x^2 + y^2 = 1$.
23. If $F(x, y, z) = y^2 i + (2xy + e^{3z})j + 3ye^{3z}k$ find a function f such that $\nabla f = F$.
24. Find the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane z = 9.

25. Compute the surface integral $\iint_S x^2\, dS$ where S is the unit sphere $x^2 + y^2 + z^2 = 1$.
26. The temperature u in a metal ball is proportional to the square of the distance from the center of the ball. Find the rate of heat flow across a sphere S of radius a with center at the center of the ball.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find the volume of the tetrahedron bounded by the planes x + 2y + z = 2, x = 2y, x = 0 and z = 0.
28. Evaluate the integral $\iint_R e^{(x+y)/(x-y)}\, dA$ where R is the trapezoidal region with vertices (1, 0), (2, 0), (0, –2) and (0, –1).
29. Using Green's theorem evaluate $\int_C (x^4 dx + xy dy)\, ds$ where C is the triangular curve consisting of the line segments from (0, 0) to (1, 0), from (1, 0) to (0, 1) and from (0, 1) to (0, 0).
30. Use Stokes' Theorem to compute the integral $\iint_S \text{curl } F . dS$ where $F(x, y, z) = xzi + yzj + xyk$ and S is the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies inside the cylinder $x^2 + y^2 = 1$ and above the xy-plane.

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