K23U 3607

Reg. No. : ....

Name : ....

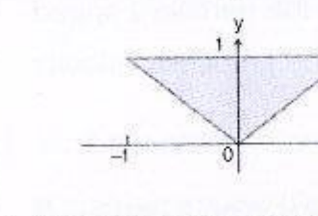
III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Supplementary) Examination, November 2023
(2018 – 2020 Admissions)
BHM302 : VECTOR CALCULUS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries **one** mark.

1. Find the gradient of the function $f(x, y) = x^2 - 2y$ at the given point $\left(\frac{1}{2}, \frac{1}{2}\right)$.
2. Describe the given region in polar coordinates.



3. Draw the graph of vector equation $r(t) = ti + (1 - t)\,j$, $0 \le t \le 1$.
4. Find the divergence of $F = 2x^2 i + 2y^2 j + 3z^2 k$.
5. State Stokes Theorem.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Find the curve $r(t) = (2 + t)\,i - (t + 1)j + tk$, $0 \le t \le 3$, unit tangent vector.
7. Let $f(x, y) = x - y$, $g(x, y) = 3y$. Find $\nabla(f - g)$.
8. Find the linearization $L(x, y)$ of the function $f(x, y) = x^2 + y^2 + 1$ at point (0,0).

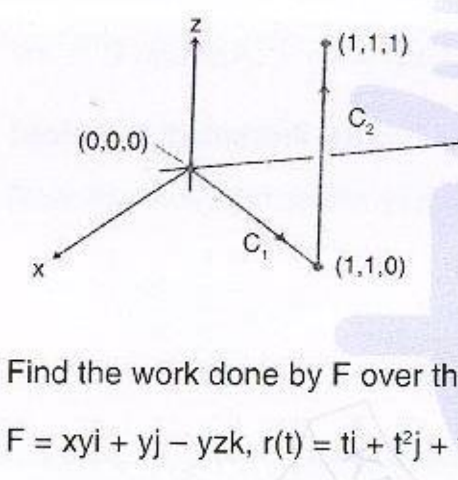
9. Evaluate the iterated integral $\int_0^2 \int_{-1}^1 (x - y)\,dy\,dx$.
10. Find a spherical coordinate equation for the sphere $x^2 + y^2 + (z-1)^2 = 1$.
11. Check whether $F = (y \sin z)\,i + (x \sin z)\,j + (xy \cos z)\,k$, is conservative or not ?
12. Find a parametrization of the sphere $x^2 + y^2 + z^2 = 4$.
13. Find the curl of $F = (x^2 - z)\,i + xe^z j + xyk$.
14. Show that curl grad f = 0.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Let $r(t) = (1+t)i + t^2 j + \frac{t^3}{3}k$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then find the particle's speed and direction of motion at the given value of t = 1. Write the particle's velocity at that time as the product of its speed and direction.
16. Find the binormal vector and torsion for the space curve $r(t) = (\cos t + t \sin t)\,i + (\sin t - t \cos t)\,j + 3k$.
17. Let $f(x, y) = x^2 - xy + y^2 - y$. Find the directions u and the values of $D_u f(1, -1)$ for which
    a) $D_u f(1, -1)$ is largest
    b) $D_u f(1, -1)$ is smallest
    c) $D_u f(1, -1) = 0$
    d) $D_u f(1, -1) = 4$.

18. Use Fubini's Theorem to evaluate $\int_0^2 \int_0^1 \frac{x}{1 + xy}\,dx\,dy$.
19. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle r = 1.
20. Find the average value of $F(x, y, z) = x + y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes x = 1, y = 1 and z = 2.
21. Integrate $f(x, y, z) = x - 3y^2 + z$ over $C_1 \cup C_2$.



22. Find the work done by F over the curve in the direction of increasing t.
    $F = xyi + yj - yzk$, $r(t) = ti + t^2 j + tk$, $0 \le t \le 1$.
23. Find a potential function for $F = \frac{2x}{y}i + \frac{1 - x^2}{y^2}j, \{(x, y) : y > 0\}$.
24. Find the area of the surface cut from the paraboloid $x^2 + y^2 - z = 0$ by the plane z = 2.
25. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes x = 1, y = 1 and z = 1.
26. Use the surface integral in Stoke's Theorem to calculate the circulation of the field $F = x^2 i + 2xj + z^2 k$ around the curve C : ellipse $4x^2 + y^2 = 4$ in the xy-plane, counterclockwise when viewed from above in the indicated direction.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find unit tangent vector, principal unit normal vector and curvature for the plane curve in $r(t) = (2t + 3)\,i + (5 - t^2)j$.
28. Find the volume of the "ice-cream cone" D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \frac{\pi}{3}$.
29. Show that $y\,dx + x\,dy + 4\,dz$ is exact and evaluate the integral
    $\int_{(1,1,1)}^{(2,3,-1)} y\,dx + x\,dy + 4\,dz$, over any path from (1, 1, 1) to (2, 3, –1).
30. Use Stoke's Theorem to evaluate $\int_C F.dr$, if $\mathbf{F} = xzi + xyj + 3xzk$ and C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counterclockwise as viewed from above.