Reg. No. : Name :

III Semester B.Sc. Honours in Mathematics Degree (CBCSS - Supplementary) Examination, November 2023 (2018 - 2020 Admissions)

BHM 301 : REAL ANALYSIS

Time: 3 Hours

Max. Marks: 60

PART - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark. (4×1=4)

- 1. Let $a \in R$ and $\epsilon > 0$. Define the ϵ -neighborhood of a.
- 2. Define the Fibonacci sequence.
- 3. When can you say that a sequence of real numbers is bounded?
- 4. When can you say that a function is piecewise linear? 5. Define rearrangement of a series.

PART-B

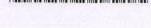
Answer any 6 questions out of 9 questions. Each question carries 2 marks. $(6 \times 2 = 12)$

- State the order properties of R.
- 7. If a.b = 0, then prove that either a = 0 or b = 0.
- 8. Prove that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.
- 9. Prove that a convergent sequence of real numbers is bounded.
- State the Cauchy criterion for series.
- 11. Prove that the geometric series diverges if $r \ge 1$.

P.T.O.

K23U 3606

-2-



 $(8 \times 4 = 32)$

- 12. Prove that the p-series $f(t) = \frac{1}{t^p}$, for $t \ge 1$ converges if p > 1 and diverges
- 13. State any two Nonuniform Continuity Criteria. 14. If $f: A \to R$ is uniformly continuous on a subset A of R and if (x_n) is a Cauchy
- sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R. PART - C

Answer any 8 questions out of 12 questions. Each question carries 4 marks.

15. Prove that there does not exist a rational number r such that $r^2 = 2$.

State and prove the Density Theorem.

prove that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

- 17. Prove that an upper bound u of a nonempty set S in R is the supremum of S if
- and only if for every \in > 0 there exists an $s_{\in} \in S$ such that $u \in < S_{\in}$. 18. Using Bisection method, find the root of the equation $f(x) = xe^x - 2 = 0$ in the
- interval [0, 1]. 19. Let I be a closed bounded interval and let $f: I \to R$ be continuous on I. Then
- 20. Let I be a closed bounded interval and let $f: I \to R$ be continuous on I. If $\epsilon > 0$, then prove that there exists a continuous piecewise linear function $g_{\varepsilon}\colon I\to R$
- such that $|f(x) g_{\in}(x)| < \epsilon$ for all $x \in I$. 21. Prove that $\lim_{n \to \infty} n^n = 1$.

 $\lim(z_n) = 0$. Then prove that the alternating series $\sum (-1)^{n+1} z_n$ is convergent.

- State and prove Squeeze Theorem. 23. Prove that the sequence S : = (sin n) is divergent.
- 24. State and prove the Comparison test for convergence of a series.
- 25. Let $Z := (z_n)$ be a decreasing sequence of strictly positive numbers with
- State and prove the Integral test for convergence.

State and prove the characterization theorem for intervals.

28. Prove that a function f is uniformly continuous on the interval (a, b) if and only

-3-

PART - D

Answer any two questions out of 4 questions. Each question carries 6 marks. (2×6=12)

K23U 3606

- if it can be defined at the endpoints a and b such that the extended function is continuous on [a, b].
- 29. Let (x_n) be a sequence of positive real numbers such that $L := \lim \left(\frac{x_n + 1}{x_n}\right)$ exists. If L < 1, then prove that (x_n) converges and $\lim (x_n) = 0$. 30. i) State and prove Abels Lemma. ii) State and prove Abels Test for convergence of series.