K23U 3606

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (CBCSS – Supplementary) Examination, November 2023
(2018 – 2020 Admissions)
BHM 301 : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Let $a \in R$ and $\in > 0$. Define the $\in$-neighborhood of a.
2. Define the Fibonacci sequence.
3. When can you say that a sequence of real numbers is bounded ?
4. When can you say that a function is piecewise linear ?
5. Define rearrangement of a series.

PART – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. State the order properties of R.
7. If $a.b = 0$, then prove that either $a = 0$ or $b = 0$.
8. Prove that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.
9. Prove that a convergent sequence of real numbers is bounded.
10. State the Cauchy criterion for series.
11. Prove that the geometric series diverges if $r \geq 1$.

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12. Prove that the p-series $f(t) = \frac{1}{t^p}$, for $t \geq 1$ converges if $p > 1$ and diverges if $p < 1$.
13. State any two Nonuniform Continuity Criteria.
14. If $f : A \to R$ is uniformly continuous on a subset A of R and if (x_n) is a Cauchy sequence in A, then prove that $(f(x_n))$ is a Cauchy sequence in R.

PART – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Prove that there does not exist a rational number r such that $r^2 = 2$.
16. State and prove the Density Theorem.
17. Prove that an upper bound u of a nonempty set S in R is the supremum of S if and only if for every $\in > 0$ there exists an $s_\in \in S$ such that $u - \in < S_\in$.
18. Using Bisection method, find the root of the equation $f(x) = xe^x - 2 = 0$ in the interval [0, 1].
19. Let I be a closed bounded interval and let $f : I \to R$ be continuous on I. Then prove that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.
20. Let I be a closed bounded interval and let $f : I \to R$ be continuous on I. If $\in > 0$, then prove that there exists a continuous piecewise linear function $g_\in : I \to R$ such that $|f(x) - g_\in(x)| < \in$ for all $x \in I$.
21. Prove that $\lim n^{\frac{1}{n}} = 1$.
22. State and prove Squeeze Theorem.
23. Prove that the sequence $S := (\sin n)$ is divergent.
24. State and prove the Comparison test for convergence of a series.
25. Let $Z := (z_n)$ be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Then prove that the alternating series $\sum(-1)^{n+1} z_n$ is convergent.
26. State and prove the Integral test for convergence.

PART – D

Answer **any two** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove the characterization theorem for intervals.
28. Prove that a function f is uniformly continuous on the interval (a, b) if and only if it can be defined at the endpoints a and b such that the extended function is continuous on [a, b].
29. Let (x_n) be a sequence of positive real numbers such that $L := \lim\left(\frac{xn+1}{x_n}\right)$ exists. If $L < 1$, then prove that (x_n) converges and $\lim (x_n) = 0$.
30. i) State and prove Abels Lemma.
    ii) State and prove Abels Test for convergence of series.

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