K23U 3601

Reg. No. : ....

Name : ....

III Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, November 2023
(2021 and 2022 Admissions)
3B09 BMH : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark.

1. State Well – Ordering Property of $\mathbb{N}$.
2. Let $S = \{1 - (-1)^n/n : n \in \mathbb{N}\}$. Find inf S and sup S.
3. Define ε – neighbourhood of a point in $\mathbb{R}$.
4. Find $\lim\left(\frac{1}{n}\right)$.
5. If the series Σx_n converges, prove that $\lim(x_n) = 0$. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Prove that the set $\mathbb{Z}$ of all integers is denumerable.
7. Using Principle of Mathematical Induction, prove that for each $n \in \mathbb{N}$, the sum of first n natural numbers is $\frac{1}{2}n(n+1)$.
8. State and prove Triangle inequality of real numbers.
9. Solve the inequality $|2x - 1| \leq x + 1$, for $x \in \mathbb{R}$.
10. Prove that a sequence in $\mathbb{R}$ can have atmost one limit.

P.T.O.

11. Prove that a Cauchy sequence of real numbers is bounded.
12. Find $\lim\left(\frac{\sin n}{n}\right)$.
13. State and prove Root test.
14. Does the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - n + 1}$ converges ? **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Prove that the set $\mathbb{N} \times \mathbb{N}$ is denumerable.
16. State and prove Cantor's theorem.
17. State and prove Archimedean property of real numbers.
18. Let S be a nonempty subset of $\mathbb{R}$ and $a \in \mathbb{R}$. Define the set $a + S = \{a + s : s \in S\}$. Prove that $\sup(a + S) = a + \sup S$.
19. State and prove Density theorem.
20. Let $I_n = [a_n, b_n]$, $n \in \mathbb{N}$ is a nested sequence of closed bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$. Prove that the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique.
21. Let $Y = (y_n)$ be defined inductively by $y_1 = 1$, $y_{n+1} = \frac{1}{4}(2y_n + 3)$, $n \in \mathbb{N}$. Find lim Y.
22. Let $X = (x_n)$ be a bounded sequence of real numbers and let $x \in \mathbb{R}$ have the property that every convergent subsequence of X converges to x. Prove that the sequence X converges to x.
23. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively. Prove that the sequence X + Y, XY converge to x + y and xy respectively.
24. State and prove Abel's test.

25. Prove that the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when p > 1.
26. Does the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ converges ? **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Suppose that S and T are sets and that $T \subseteq S$. If S is a finite set, then prove that T is a finite set.
28. Prove that the set $\mathbb{R}$ of real numbers is not countable.
29. State and prove monotone subsequence theorem.
30. State and prove Raabe's test. **(2×6=12)**