Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2023
(2017-2020 Admission)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define a binary operation.
2. Give an example of a cyclic group.
3. Write an example of an infinite dimensional vector space.
4. What is the dimension of $\mathbb{R}^2$ over $\mathbb{R}$?
5. Give an example of a linear map $T : \mathbb{R} \to \mathbb{R}$. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Give an example of a subset of $\mathbb{Z}$ which is not a subgroup. Justify.
7. Show that in a group G, $(ab)^{-1} = b^{-1} a^{-1}$.
8. Write all the subgroups of the group $\mathbb{Z}_6$.
9. List all the generators of the group $\mathbb{Z}_{10}$.
10. Find the order of the permutation (1, 3, 6), (2, 8), (4, 7, 5).
11. Define subspace of a vector space.

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12. Show that the set {(1, 0), (0, 1)} is linearly independent.
13. Show that the set $W = \{(x, y) \in \mathbb{R}^2 : x + y = 0\}$ is subspace of $\mathbb{R}^2$.
14. Define null space and range space of a linear transformation. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Let $G = \{q \in \mathbb{Q} : q \neq -1\}$. Define the binary operation $*$ on G by $x * y = x + y + xy$. Show that $(G, *)$ is an abelian group.
16. Write all abelian subgroups of S_3.
17. Find all subgroups of the group $\mathbb{Z}_6$. Give the group table of the group.
18. Show that the group of positive rational numbers under multiplication is not cyclic.
19. Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$. Compute a^{-1} and $\alpha\beta$.
20. Describe the group of symmetry of a triangle.
21. Show that the set $W = \{(x, y, z) : x + y - z = 0\}$ is a subspace of $\mathbb{R}^3$. Find its dimension.
22. Show that the vectors $v_1 = \{1, 2, 1\}$, $v_2 = \{2, 9, 0\}$ and $v_3 = \{3, 3, 4\}$ form a basis of $\mathbb{R}^3$.
23. Write a basis for $\mathbb{R}^2$ over $\mathbb{R}$. Express (1, 2) as the linear combination of the basis.
24. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y, z) = (x + y, x - y, z)$, show that T is a linear transformation.

25. Show that a linear map T in a finite dimensional vector space is one-one if and only if N(T) = {0}.
26. Let T be the linear transformation form $\mathbb{R}^3$ to $\mathbb{R}^2$ defined by $T(x, y, z) = (x + y, 2z - x)$. Find matrix representation of T with respect to the standard basis. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Prove that a group G is abelian if and only if $(ab)^{-1} = a^{-1} b^{-1}$.
28. Give the group table of the group S_3.
29. Show that if a vector space V has a dimension n, then any vectors of n + 1 elements is linearly dependent.
30. Let V be the space of all real polynomials of degree less than or equal to 3, and D be the differentiation operator on V. Find [D] associated with the bases $\{1, x, x^2, x^3\}$. **(2×6=12)**