K23U 2022

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (CBCSS – OBE – Regular/ Supplementary/Improvement) Examination, April 2023
(2021 and 2022 Admission)
2B05BMH : CALCULUS – II

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark.

1. Find the parametric equation for the circle with the center (h, k) and radius r.
2. Write down the formula for finding the length of the arc for the curve $x = f(t)$, $y = g(t)$ from the point at which $t = \alpha$ to the point where $t = \beta$.
3. Briefly explain the meaning of saying $\lim_{n \to \infty} a_n = \infty$.
4. State the limit comparison test for $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ where $a_n > 0$ and $b_n > 0$.
5. Write down the general equation of a quadric surface. **(4×1=4)**

SECTION – B

Answer **any 6** out of 9 questions. **Each** question carries **2** marks.

6. Find the slope of the tangent to the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$ at $\theta = \pi/3$.
7. Represent the point with the Cartesian co-ordinate (1, –1) in terms of polar co-ordinates.
8. If $\lim_{n \to \infty} |a_n| = 0$, show that $\lim_{n \to \infty} a_n = 0$.
9. Show that $\sum_{n=1}^{\infty} \frac{n^2}{5n^2 + 4}$ diverges by giving justification for your claim.

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10. Show that every absolutely convergent series is convergent.
11. Find the equation of the plane that passes through the points P(1, 3, 2), Q(3, –1, 6) and R(5, 2, 0).
12. Find the unit tangent vector at t = 0 on the space curve $\vec{r}(t) = (1 + t^2)\vec{i} + te^{-1}\vec{j} + \sin 2t\vec{k}$.
13. Find $\frac{\partial z}{\partial x}$ if $x^3 + y^3 + z^3 + 6xyz = 1$.
14. Find $D_u f(1, 2)$ if $f(x, y) = x^3 - 3xy = 4y^2$ and $u = \cos\frac{\pi}{6}\vec{i} + \sin\frac{\pi}{6}\vec{j}$. **(6×2=12)**

SECTION – C

Answer **any 8** out of 12 quetions. **Each** question carries **4** marks.

15. Find the surface area of a sphere of radius r using integration.
16. Find the limit of the sequence $\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \ldots\right\}$.
17. For what values of p is the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ convergent. Prove your claim in detail.
18. Show that the series $\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^5}}$ diverges.
19. Find the Maclaurin's series for $f(x) = \frac{1}{\sqrt{4 - x}}$. What is the radius of convergence for this series of f(x) ?
20. Find the vector function that represents the curve of intersection of the surfaces $x^2 + y^2 = 1$ and $y + z = 2$.
21. Find the directional derivative of $f(x, y, z) = x \sin yz$ at (1, 3, 0) in the direction of $\vec{v} = \vec{i} = 2\vec{j} - \vec{k}$.
22. Find the shortest distance from (1, 0, –2) to the plane $x + 2y + z = 4$.

23. Find the curvature of the twisted cubic $r(t) = (t, t^2, t^3)$ at the origin.
24. Show in detail that $\lim_{(x,y) \to (0,0)} \frac{3x^2y}{x^2 + y^2} = 0$.
25. Using the method of linearization, find an approximate value of f(1.1, –0.1) if $f(x, y) = xe^{xy}$.
26. If $g(r, s, t) = f(r - s, s - t, t - r)$, show that $r\frac{\partial g}{\partial r} + s\frac{\partial g}{\partial s} + t\frac{\partial g}{\partial t} = 0$. **(8×4=32)**

SECTION – D

Answer **any 2** out of 4 questions. **Each** question carries **6** marks.

27. a) Suppose the temperature at a point (x, y, z) in space is given by $T(x, y, z) = \frac{80}{1 + x^2 + 2y^2 + 3z^2}$. In what direction does the temperature increases fast at the point (1, 1, –2) and what is that maximum rate of increase ?

    b) Find the radius of convergence and the interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{(-3x)^n}{\sqrt{n+1}}$.
28. The dimensions of a rectangular box are measured to be 75 cms, 60 cms and 40 cms and each measurement is correct within 0.2 cms. Use differentials to estimate the largest possible error when the volume of the box is calculated from these measurements.
29. a) Find the equations of tangent plane and normal line at the point P(–2, 1, –3) to the ellipsoid $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 3$.

    b) Find the local maximum, local minimum and the saddle points, if any for the function $f(x, y) = x^4 + y^4 - 4xy + 1$.
30. a) Using Lagrange multiplier method find the extreme values of $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

    b) Find the length of one arch of the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$. **(2×6=12)**

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