Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – OBE – Regular/Supplementary/Improvement) Examination, April 2023
(2021 and 2022 Admission)
2B06 BMH : DISTRIBUTION FUNCTIONS AND COMBINATORICS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of five questions. **Each** question carries **1** mark.

1. Describe Bernoulli Distribution.
2. Give the characteristic function of the Poisson Distribution.
3. Describe Normal Distribution.
4. If $m, n \in \mathbb{Z}^+$, then $\phi(n^m) =$ ____________
5. $\frac{x+1}{(1-x)^2} =$ ____________ **(4×1=4)**

SECTION – B

Answer **any six** questions out of nine questions. **Each** question carries **2** marks.

6. A and B play a game in which their chances of winning are in the ratio 3 : 2. Find A's chance of winning at least three games out of the five games played.
7. If $X \sim B(n, p)$, evaluate $E\left(\frac{X}{n}-p\right)^2$.
8. Describe rectangular distribution.
9. Give three properties of normal probability curve.
10. X is normally distributed and the mean of X is 12 and S.D. is 4. Find out the probability of $X \geq 20$.

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11. List the nine derangements of 1, 2, 3 and 4.
12. If there is an unlimited number (or at least 24 of each colour) of red, green, white and black jelly beans, in how many ways can Douglas select 24 of these candies so that he has an even number of white beans and at least six black ones.
13. Find the coefficient of x^7 in $(1 + x + x^2 + x^3 + ...)^n$, $n \in \mathbb{Z}^+$.
14. Find the exponential generating function for the sequence $1, a^2, a^4, a^6, ...$ $a \in \mathbb{R}$. **(6×2=12)**

SECTION – C

Answer **any eight** questions out of twelve questions. **Each** question carries **4** marks.

15. An irregular six faced die is thrown and the expectation that in 10 throws it will give five even numbers is twice the expectation that it will give four even numbers. How many times in 10,000 sets of 10 throws each would you expect it to give no even number ?
16. Find μ_1' of the Poisson distribution.
17. Suppose X is a non-negative integral valued random variable. Show that the distribution of X is geometric if for each $k \geq 0$ and $Y = X - k$ one has $P(Y = t|X \geq k) = P(X = t)$, for $t \geq 0$.
18. If X is uniformally distributed with mean 1 and variance $\frac{4}{3}$, find $P(X < 0)$.
19. Find the mean deviation from the mean for normal distribution.
20. If X is a standard normal variate, find $E|X|$.
21. Six married couples are to be seated at a circular table. In how many ways can they arrange themselves so that no wife sits next to her husband ?
22. While at the racetrack, Ralph bets on each of the ten horses in a race to come in according to how they are favored. In how many ways can they reach the finish line so that he loses all of his bets ?

23. Ten women attend a business luncheon. Each woman checks her coat and attache case. Upon leaving, each woman is given a coat and case at random. In how many ways can they be distributed so that no woman gets back both of her possessions ?
24. In how many ways can a police captain distribute 24 rifle shells to four police officers so that each officer gets at least three shells, but not more than eight ?
25. Find a formula to express $0^2 + 1^2 + 2^2 + ... + n^2$ as a function of n.
26. A ship carries 48 flags, 12 each of the colors red, white, blue, and black. Twelve of these flags are placed on a vertical pole in order to communicate a signal to other ships. How many of these signals use an even number of blue flags and an odd number of black flags ? **(8×4=32)**

SECTION – D

Answer **any two** questions out of four questions. **Each** question carries **6** marks.

27. A department in a works has 10 machines which may need adjustment from time to time during the day. Three of these machines are old, each having a probability of 1/11 of needing adjustment during the day, and 7 are new, having corresponding probabilities of 1/21. Assuming that no machine needs adjustment twice on the same day, determine the probabilities that on a particular day.
    a) just 2 old and no new machines need adjustment.
    b) If just 2 machines need adjustment, they are of the same type.
28. If X_1 and X_2 are independent rectangular variates on [0, 1], find the distributions of X_1/X_2 and $X_1 + X_2$.
29. Consider a set S, with $|S| = N$, and conditions c_i, $1 \leq i \leq t$, each of which may be satisfied by some of the elements of S. Then prove that the number of elements of S that satisfy exactly m of the conditions c_i, $1 \leq i \leq t$, is given by
    $$E_m = S_m - \binom{m+1}{1}S_{m+1} + \binom{m+2}{2}S_{m+2} + ... + (-1)^{t-m}\binom{t}{t-m}S_t.$$
30. We have a pair of dice; one is red, the other green. We roll these dice six times. What is the probability that we obtain all six values on both the red die and the green die if we know that the ordered pairs (1, 2), (2, 1), (2, 5), (3, 4), (4, 1), (4, 5) and (6, 6) did not occur ? **(2×6=12)**