K23U 2018

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2023
(2017-2020 Admission)
BHM 203 : Integral Calculus

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Find a formula for the n^{th} term of the sequence 1, 5, 9, 13,17, ...
2. Given $a_n = \frac{1-n}{n^2}$. Find a_1, a_2, a_3, a_4.
3. Find the sum of the geometric series $\sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{1}{3}\right)^{n-1}$.
4. Give the formula for the work done by a force F(x) along the x – axis from x = a to x = b.
5. State the mean value theorem for definite integrals.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find a power series representation of $f(x) = \ln(1 + x), -1 < x \le 1$.
7. Find the Taylor series and Taylor polynomials generated by $f(x) = e^x$ at $x = 0$.
8. Express the solution of the initial value problem $\frac{dy}{dx} = \sec x, y(2) = 3$.
9. Express the limit $\lim_{\|P\| \to 0} \sum_{k=1}^{n} (c_k^2 - 3c_k)\Delta x_k$, where P is a partition of [–7, 5].
10. Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot\theta \operatorname{cosec}^2\theta \, d\theta$.

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11. Find the area between $y = \sec^2 x$ and $y = \tan^2 x, x = -\frac{\pi}{4}, x = \frac{\pi}{4}$.
12. The region between the curve $y = \sqrt{x}, 0 \le x \le 4$ and the x – axis is revolved about the x – axis to generate a solid. Find the volume.
13. Using reduction formula, evaluate $\int \sin^2 x \, dx$.
14. Using reduction formula, evaluate $\int \tan^6 x \, dx$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the Maclaurin series for cos x.
16. Find the sum of $\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}}$.
17. Find the Taylor series expansion at x = 0, of $e^x \cos x$.
18. Find the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1, 0 \le x \le 1$.
19. The line segment $x = 1 - y, 0 \le y \le 1$, is revolved about the y – axis to generate a cone. Find its lateral surface area.
20. Find the center of mass of a wire of constant density δ shaped like a semicircle of radius a.
21. A spring has a natural length of 1 m. A force of 24 N stretches the spring to a length of 1.8 m.
    a) Find the force constant k.
    b) How far will a 45 N force stretch the spring ?
22. Evaluate $\int_0^1 \sqrt{t^5 + 2t}(5t^4 + 2)\, dt$ and $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} (1 - \cos 3t)\sin 3t \, dt$.
23. Show that $\frac{(-1)^{n+1}(n-1)}{n}$ diverges.
24. State the n^{th} term test for divergence and hence test the divergence of the series $\sum_{n=1}^{\infty} n^2$.

25. State second part of the Fundamental Theorem of Calculus and evaluate $\int_0^{\pi} \cos x \, dx$.
26. Show that if f is continuous on [a, b], a ≠ b, and if $\int_a^b f(x)\, dx = 0$ then f(x) = 0 atleast once in [a, b].

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** carry **6** marks. **(2×6=12)**

27. Find the area of the region that lies inside the circle r = 1 and outside the cardioid $r = 1 - \cos\theta$.
28. Find the area of the surface generated by revolving the right hand loop of the lemniscate $r^2 = \cos 2\theta$ about the y – axis.
29. Give the reduction formula for $\int_0^{\frac{\pi}{2}} \cos^n x \, dx$ and evaluate $\int_0^{\infty} \frac{1}{(1+x^2)^n} dx$.
30. a) Show that the value of $\int_0^1 \sqrt{1 + \cos x}\, dx$ cannot possibly be 2.
    b) Use the inequality $\cos x \ge \left(1 - \frac{x^2}{2}\right)$, which holds for all x, to find a lower bound for the value of $\int_0^1 \cos x \, dx$.

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