K23U 2024

Reg. No.: ....

Name

II Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – OBE – Regular/Supplementary/Improvement) Examination, April 2023
(2021 and 2022 Admission)
2B07 BMH : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any four** questions out of **five** questions. **Each** question carries **1** mark.

1. Define repunit.
2. State True/False : $12 \equiv 3 \pmod 5$.
3. Find $\tau(10)$.
4. Find all roots of $f(x) = x^2 - x - 6$.
5. Define reciprocal equation. **(4×1=4)**

SECTION – B

Answer **any six** questions out of nine questions. **Each** question carries **2** marks.

6. Show that the square of an integer leaves the remainder 0 or 1 upon division by 4.
7. For integers a, b, c, prove the following :
   If $a|b$ and $a|c$, then $a|(bx + cy)$ for arbitrary integers x and y.
8. Using Euclidean Algorithm calculate gcd(12378, 3054).
9. If p is a prime and $p|ab$, then prove that $p|a$ or $p|b$.
10. List first five Euclidean numbers.

P.T.O.

11. Let $n > 1$ be fixed and a, b, c be arbitrary integers. Then prove the following :
    If $a \equiv b \pmod n$ and $b \equiv c \pmod n$, then $a \equiv c \pmod n$.
12. Find the remainder obtained when 5^{38} is divided by 11.
13. If α, β, γ are the roots of $x^3 - x - 1 = 0$, find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}$ and $\frac{1+\gamma}{1-\gamma}$.
14. Find the sum of the squares of roots of the equation $x^3 - px^2 + qx - r = 0$. **(6×2=12)**

SECTION – C

Answer **any eight** questions out of twelve questions. **Each** question carries **4** marks.

15. Given integers a and b, not both of which are zero, then prove that there exist integers x and y such that $\gcd(a, b) = ax + by$.
16. If $a|bc$, with $\gcd(a, b) = 1$ then prove that $a|c$.
17. If a cock is worth 5 coins, a hen 3 coins, and three chicks together 1 coin, how many cocks, hens and chicks, totaling 100, can be bought for 100 coins ?
18. If $d|c$, where $d = \gcd(a, b)$, then prove that the linear Diophantine equation $ax + by = c$ has a solution.
19. Prove that there is an infinite number of primes.
20. Using Sieve of Eratosthenes find all primes not exceeding 100.
21. If p_n is the nth prime number, then prove that $p_n \le 2^{2^{n-1}}$.
22. Show that 41 divides $2^{20} - 1$.
23. Solve the linear congruence $17x \equiv 9 \pmod{276}$.
24. Find the number and sum of divisors of 21600.

25. Solve the reciprocal equation
    $60x^4 - 736x^3 + 1433x^2 - 736x + 60 = 0$.
26. Determine the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$. **(8×4=32)**

SECTION – D

Answer **any two** questions out of **four** questions. **Each** question carries **6** marks.

27. Prove that the linear congruence $ax \equiv b \pmod n$ has a solution if and only if $d|b$, where $d = \gcd(a, n)$. If $d|b$, then prove that it has d mutually incongruent solutions modulo n.
28. State and prove Wilson's Theorem.
29. If $n = p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$ is the prime factorization of $n > 1$, then prove that
    a) $\tau(n) = (k_1 + 1)(k_2 + 1) \ldots (k_r + 1)$, and
    b) $\sigma(n) = \frac{p_1^{k_1+1} - 1}{p_1 - 1} \frac{p_2^{k_2+1} - 1}{p_2 - 1} \cdots \frac{p_r^{k_r-1} - 1}{p_r - 1}$.
30. Solve the equation $x^4 - 4x^3 - 10x^2 + 64x + 40 = 0$. **(2×6=12)**

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