K23U 2019

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Supple.)
Examination, April 2023
(2017 – 2020 Admission)
BHM 204 : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Any 4 out of 5 questions. **Each** question carries **1** mark.

1. For integers a, b, c prove that if a | b and b | c then a | c.
2. Define the least common multiple of two integers a and b.
3. Determine whether the Diophantine equation $33x + 14y = 115$ is solvable.
4. State Remainder theorem.
5. State Descartes, Rule of signs for positive roots.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Prove that a and b are relatively prime if and only if there exist integers x and y such that $ax + by = 1$.
7. For integers, a, b and c if a|b and $b \neq 0$, then prove that $|a| \leq |b|$.
8. If a and b are given integers, not both zero, then prove that the set $T = \{ax + by \mid x, y \text{ are integers}\}$ is precisely the set of all multiples of d=gcd(a, b)
9. If a|c and b|c, with gcd(a, b) = 1, then ab | c.
10. Solve the linear congruence $18x \equiv 30 \pmod{42}$
11. Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one of the roots is $1 - \sqrt{5}$.

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12. If $x_1, x_2, ...x_n$ are the roots of the equation $(a_1 - x)(a_2 - x)...(a_n - x) + k = 0$, then show that $a_1, a_2, ...a_n$ are the roots of the equation $(x_1 - x)(x_2 - x) ... (x_n - x) - k = 0$.
13. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ of which one root is $-1 + \sqrt{-1}$.
14. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, express the value $\Sigma \alpha^2 \beta$ of in terms of the coefficients.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Solve the linear Diophantine equation $172x + 20y = 1000$.
16. Prove that $a \equiv b \pmod n$ if and only if a and b leave the same non negative remainder when divided by n.
17. Solve the system of three congruences $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 2 \pmod 7$.
18. If $ca \equiv cb \pmod n$, then $a \equiv b \left(\text{mod} \frac{n}{d}\right)$ where d=gcd(c, n).
19. If p is a prime and $p \mid a_1 a_2 a_3 ... a_n$ then $p \mid a_k$ for some k, $1 \leq k < n$.
20. If α be a real root of the cubic equation $x^3 + px^2 + qx + r = 0$, of which the coefficients are real, show that the other two roots of the equation are real if $p^2 \geq 4q + 2pa + 3a^2$.
21. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.
22. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.
23. Find the number of real roots of the equation $x^4 - 14x^2 + 16x + 9 = 0$.
24. Find the roots of the equation $x^5 + 4x^4 + 3x^3 + 3x^2 + 4x + 1 = 0$.
25. If α, β, γ be the roots of the equation $x^3 - 6x + 7 = 0$, form an equation whose roots are $\alpha^2 + 2\alpha + 3, \beta^2 + 2\beta + 3, \gamma^2 + 2\gamma + 3$.
26. Find the condition that the cubic equations $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots, and when the condition is satisfied, find the equal roots.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Prove that the linear Diophantine equation $ax + by = c$ has a solution if and only if d|c, where d=gcd(a,b). If x_0, y_0 is any particular solution of this equation, then all other solutions are given by $x = x_0 + \left(\frac{b}{d}\right)t, y = y_0 - \left(\frac{a}{d}\right)t$.
28. State and prove Chinese Remainder Theorem.
29. Show that the roots of the equation $x^3 + px^2 + qx + r = 0$ are in Arithmetical progression if $2p^3 - 9pq + 27r = 0$.
30. If $\alpha, \beta, \gamma, \delta$ be the roots of the biquadratic equation $x^4 + px^3 + qx^2 + rx + s = 0$, find
    1) $\Sigma \alpha^2$
    2) $\Sigma \alpha^2 \beta\gamma$
    3) $\Sigma \alpha^2 \beta^2$
    4) $\Sigma \alpha^3 \beta$
    5) $\Sigma \alpha^4$.

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