K23U 4126

Reg. No. : ……………………………

Name : …………………………………

I Semester B.Sc. Honours in Mathematics (CBCSS – OBE-Regular/ Supplementary/Improvement) Examination, November 2023
(2021 to 2023 Admissions)
CORE COURSE
1B04 BMH : Two Dimensional Geometry

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Write the general form of second degree equation in x and y.
2. Write the parametric equation of the parabola $y^2 = 4ax$.
3. Define asymptote.
4. What is the length of latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
5. Find the polar equation of the conic with focus as pole and axis of the conic as initial line.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Simplify the equation $x^2 + 2y^2 - 6x + 16y + 39 = 0$ by changing to a new origin (3, –4).
7. Find the angle between the lines $x^2 + 3xy + 2y^2 = 0$.
8. Show that the equation $x^2 + 8xy + y^2 + 16x + 4y + 4 = 0$ represents a pair of lines.
9. Find the equation of the parabola whose vertex is (3, 3) and focus is (–3, 3).

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10. Prove that the condition for the line $y = mx + c$ touches the parabola $y^2 = 4ax$ is $c = \frac{a}{m}$.
11. Find the eccentricity and foci of the ellipse $4x^2 + 9y^2 = 144$.
12. Prove that four normals can be drawn to the hyperbola $xy = c^2$ from any given point.
13. Find the asymptotes of the conic $x^2 - 3xy + y^2 + 10x - 10y + 21 = 0$.
14. Prove that the equation of the chord joining two points on a conic $\frac{l}{r} = 1 + e\cos\theta$ is $\frac{l}{r} = \sec\beta\cos(\theta - \alpha) + e\cos\theta$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Transform the equation $x^2 - 4xy + y^2 + 8x + 2y - 5 = 0$ to a new axes through (2, 3) rotated through an angle of 45°.
16. Find the equation of the bisectors of the angles between the lines represented by $3x^2 + 8xy + 4y^2 = 0$.
17. Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may represent a pair of lines.
18. Prove that the locus of poles of normal chords of the parabola $y^2 = 4ax$ is $(x + 2a)y^2 + 4a^3 = 0$.
19. Prove that the tangents at the extremities of a focal chord of a parabola intersect right angles on the directrix.
20. Find the locus of the point of intersection of normals at the ends of a focal chord of the parabola $y^2 = 4ax$.
21. A variable chord substends a right angle at the centre of the ellipse. Find the locus of the point of intersection of the tangents at the ends.
22. Prove that from any point four normals can be drawn to an ellipse.

23. Prove that the locus of poles of normal chords with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is the curve $\frac{a^6}{x^2} - \frac{b^6}{y^2} = (a^2 + b^2)^2$.
24. Prove that the coordinates of the centre of a conic section is $\left(\frac{fh - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$.
25. Find the equation of the directrix and focus of the parabola $x^2 - 2xy + y^2 - 2x - 2y + 3 = 0$.
26. If PSP′ and QSQ′ be any two focal chords of a conic which are at right angles to one another, prove that $\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$ is constant.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of lines, prove that the equation to the pair of lines passing through the points where these meet the axes is $ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \frac{4fg}{c}xy = 0$.
28. Show that the equation of the pair of tangents drawn from an external point $P(x_1, y_1)$ to the parabola $y^2 = 4ax$ is $T^2 = SS_1$, where $S \equiv y^2 - 4ax$, $S_1 \equiv y_1^2 - 4ax_1$ and $T \equiv yy_1 - 2ax - 2ax_1$.
29. If CP and CD are conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Prove that the locus of the orthocenter of the triangle CPD is the curve $2(b^2y^2 + a^2x^2)^3 = (a^2 - b^2)^2(b^2y^2 - a^2x^2)^2$.
30. Trace the conic $9x^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0$.

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