



Reg. No. :

Name :

I Semester B.Sc. Honours in Mathematics (CBCSS – OBE – Regular/
Supplementary/Improvement) Examination, November 2023
(2021 to 2023 Admissions)
CORE COURSE
1B01 BMH : Calculus – I

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark.

1. Define the exponential function.
2. Is the function $f(x) = x^3$ is one-one. Justify your answer.
3. Show that $\sinh(-x) = -\sinh(x)$.
4. If it is known that $\int_0^{10} f(x)dx = 17$ and $\int_0^8 f(x)dx = 12$, find $\int_8^{10} f(x)dx$.
5. Find $\int xe^x dx$.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks.

6. Find $\int \sec x dx$.
7. Find the moments of the systems of objects that have masses 4, 2 and 4 at the points (2, -3), (-3, 1) and (3, 5).

P.T.O.

K23U 4123

-2-



8. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.
9. Find the average value of the function $f(x) = 1 + x^2$ on the interval $[-1, 2]$.
10. Evaluate $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x}$.
11. State the Mean Value Theorem.
12. Find the critical numbers of the function $f(x) = x^2(1 - x)$.
13. If f and g are continuous at a , then show that fg is also continuous at a .
14. Find the inverse of the function $f(x) = x^3 + 2$.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks.

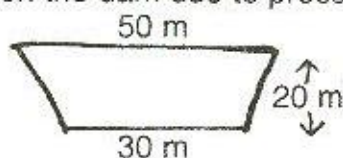
15. A bacteria culture starts with 500 bacteria and doubles in size every half hour.
 - a) How many bacteria there after 3 hours ?
 - b) How many bacteria there after t hours ?
 - c) How many bacteria there after 40 minutes ?
16. a) Find the domain of the function $f(x) = \ln(e^x - 3)$.
b) Show that $\cos(\tan^{-1} x) = \frac{1}{\sqrt{1+x^2}}$.
17. Using $\epsilon - \delta$ definition to show that $\lim_{x \rightarrow -1} x^2 = 1$.
18. Find the linearization of the function $f(x) = \sqrt{x+3}$ at 1 and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$.
19. Show that $\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1})$, $x \geq 1$.
20. Verify that the function $f(x) = x^3 - 3x + 2$, satisfies the hypothesis of the Mean Value Theorem on the interval $[-2, 2]$. Then find all numbers that satisfy the conclusion of the mean value theorem.



-3-

K23U 4123

21. Find the area enclosed between the parabolas $y^2 = x$ and $x^2 = y$.
22. Show that the volume of a sphere of radius r is $V = \frac{4}{3} \pi r^3$.
23. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ m/s.
 - a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.
 - b) Find the distance travelled during this time period.
24. Evaluate $\int_0^3 \frac{dx}{x-1}$ if possible.
25. A dam has the shape of a trapezoid shown in the following figure. The height is 20 m and the width is 50 m at the top and 30 m at the bottom. Find the force on the dam due to pressure if the water level is 4 m from the top of the dam.



26. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \cos^5 x dx$.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

27. Define the centroid of a region and find the centroid of the region bounded by the line $y = x$ and the parabola $y = x^2$.
28. a) A force of 40 N is required to hold a spring that has been stretched from its natural length 10 cm to 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm ?
b) Find $\frac{d}{dx} \int_1^x \sec t dt$.
29. Discuss the curve $f(x) = \frac{\cos x}{2 + \sin x}$ with respect to local maxima and minima, points of inflexion and concavity.
30. Find the vertical asymptotes and horizontal asymptotes of the curve $y = \frac{2x^2 + x - 1}{x^2 + x - 2}$.