K23U 4124

Reg. No. : ....

Name : ....

I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – OBE – Regular/Supplementary/Improvement) Examination, November 2023 (2021 to 2023 Admissions) Core Course

1B02 BMH : FOUNDATIONS OF MATHEMATICS

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define an embedding of X in Y.
2. Show that $f(x) = x^2$ is decreasing in $(-\infty, 0]$.
3. What do you mean by the base case in mathematical induction ?
4. Define the length of a vector.
5. When can you say that a system of linear equations is said to be consistent ?

PART – B

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

6. Let $f : Z \to Z$ and $g : N \to Z$ defined by $f(x) = g(x) = x^2$. Is $f = g$? Justify.
7. Show that the circle $\{(x, y) \in R^2 : x^2 + y^2 = 1\}$ is not graph of any function.
8. Check whether the function $f(x) = x^2$ defined on R is one-one or not.
9. Differentiate the Induction Principle and the Strong Induction Principle.
10. Define the greatest common divisor of two integers. Explain with an example.

P.T.O.

11. Calculate a^Tb and ab^T for $a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, b = \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}$.
12. Prove that if $\langle n, v \rangle = 0$ and $\langle n, w \rangle = 0$ then $\langle n, sv + tw \rangle = 0$, for any $s, t \in R$.
13. State the elementary row operations of a matrix.
14. Show that the linear system $x + y + z = 6, x + y + z = 1$ is inconsistent.

PART – C

Answer **any 8** questions from this Part. **Each** question carries **4** marks. **(8×4=32)**

15. Define max{f, g} and min{f, g}. Plot the graph of max{f, g} and min{f, g}, where $f(x) = \sin x$; $g(x) = \cos x$ defined on $[-2\pi, 2\pi]$.
16. Show that the function $f : [0, \pi) \to R$ given by $f(x) = \cos x$ is one-one.
17. Let $f : X \to Y$ and $g : Y \to Z$ be functions. Then prove that,
    a) If f and g are injective (one-one), then $g \circ f$ is injective.
    b) If f and g are surjective (onto), then $g \circ f$ is surjective.
18. For any positive integer n, we show that $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$.
19. For $n \in N$ define an as follows : $a_1 = 1$, $a_2 = 8$ and $a_n = a_{n-1} + 2a_{n-2}$ for $n \geq 3$. Prove that for each $n \in N$, $a_n = 3.2^{n-1} + 2(-1)^n$.
20. Prove that every amount of postage that is at least 12 rupees can be made from 4-rupee and 5-rupee stamps.
21. Let $a, b \in R^2$ and let θ denote the angle between them. Then show that $\langle a, b \rangle = \|a\| \|b\| \cos\theta$.

22. Prove that the equations $x = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $x = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} + t \begin{pmatrix} -3 \\ -6 \\ 3 \end{pmatrix}$ s, $t \in R$ describe the same line.
23. Find a vector equation of the line through the points A = (4, 5, 1) and B = (1, 3, –2). Find values of c and d such that the points A, B and C = (c, d, –5) are collinear.
24. Using Gauss elimination method, solve,
    $x_1 + x_2 + x_3 = 3$
    $2x_1 + x_2 + x_3 = 4$
    $x_1 - x_2 + 2x_3 = 5$.
25. If A is an $m \times n$ matrix with $m < n$ then prove that $Ax = 0$ has infinitely many solutions.
26. Let $E = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Write down E^{-1}. Then show that $EE^{-1} = I$ and $E^{-1}E = I$.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

27. Prove that a function $f : X \to Y$ is one-one if and only if $A = f^{-1}(f(A))$ for each $A \subseteq X$.
28. Let a, b be integers, not both zero, and d be the greatest common divisor of a and b. Then prove that there exist integers x, y such that $d = ax + by$.

29. Find a Cartesian equation of the plane given by
    $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$, $s, t \in R$. Show that the equation
    $x = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + s \begin{pmatrix} 6 \\ 2 \\ -5 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ -7 \end{pmatrix}$, $s, t \in R$ represents the same plane.
30. Solve the following system of equations $Ax = b$ by reducing the augmented matrix to reduced row echelon form :
    $x_1 - x_2 + x_3 + x_4 + 2x_5 = 4$
    $-x_1 + x_2 + x_4 - x_5 = -3$
    $x_1 - x_2 + 2x_3 + 3x_4 + 4x_5 = 7$.
    Show that your solution can be written in the form $x = p + sv_1 + tv_2$ where $Ap = b$, $Av_1 = 0$ and $Av_2 = 0$.