K23U 4129

Reg. No. : ....

Name : ....

I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – Supplementary/ One Time Mercy Chance) Examination, November 2023
(2016-2020 Admissions)
BHM 103 : DIFFERENTIAL CALCULUS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. State sandwich theorem.
2. Define limit of a function.
3. Find the n^{th} derivative of e^{ax}.
4. Evaluate $\lim_{x \to 0} \frac{3x - \sin x}{x}$.
5. Find $\frac{\partial f}{\partial y}$ as a function if $f(x, y) = y \sin xy$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.
7. Find $\lim_{x \to \infty} \frac{2x + 3}{5x + 7}$.
8. At what points do the graph of $f(x) = 4x^2 + 2x - 5$ has horizontal tangent lines.
9. Determine the concavity of $y = 3 + \sin x$ on $[0, 2\pi]$.

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10. If $y = \sin(ax + b)$, prove that $y_n = a^n \sin\left[\frac{n\pi}{2} + (ax + b)\right]$.
11. Find $\int_0^{\frac{\pi}{6}} \tan 2x \, dx$.
12. Solve the equation $e^{2x-6} = 4$ for x.
13. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ of $f(x, y) = \frac{1}{x + y}$.
14. Prove that $f(x, y, z) = x^2 + y^2 - 2z^2$ satisfies Laplace equation.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Show that $\lim_{x \to 1} (5x - 3) = 2$.
16. If f has a derivative at x = c, prove that f is continuous at x = c.
17. State mean value theorem and verify mean value theorem for $f(x) = x^3 - x^2$ on $[-1, 2]$.
18. A particle is moving along a horizontal coordinate line with position function $s(t) = 2t^3 - 14t^2 + 22t - 5, t \geq 0$. Find the velocity and acceleration and describe the motion of the particle.
19. A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area the rectangle can have, and what are its dimensions ?
20. Find the n^{th} derivative of $\frac{1}{x^2 + a^2}$.
21. Find $\frac{dy}{dx}$ if $y = \frac{(x^2 + 1)(x + 3)^{1/2}}{x - 1}$, $x > 1$.
22. Apply L'Hopital's rule to show that $\lim_{x \to 0^-} (1 + x)^{1/x} = e$.

23. Evaluate $\int_0^1 \frac{2}{\sqrt{3 + 4x^2}} dx$.
24. Find $\lim_{(x,y) \to (0,0)} \frac{4xy^2}{x^2 + y^2}$ if it exists.
25. Express $\frac{\partial \omega}{\partial r}$ and $\frac{\partial \omega}{\partial s}$ in terms of r and s if $w = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$.
26. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Let $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$
    i) Find the critical points of f.
    ii) Identify the open intervals on which f is increasing and decreasing.
    iii) Find the local and absolute extreme values of f.
28. If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ and hence show that $(1 - x^2) y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$.
29. One model for the way diseases die out when properly treated assumes that the rate $\frac{dy}{dt}$ at which the number of infected people changes is proportional to the number y. The number of people cured is proportional to the number y that are infected with the disease. Suppose that in the course of any given year the number of cases of a disease is reduced by 20%. If there are 10,000 cases today, how many years will it take to reduce the number to 1000 ?
30. Find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$.

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