Reg. No. : ....

Name : ....

I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – OBE – Regular/ Supplementary/Improvement) Examination, November 2023
(2021 to 2023 Admissions)
Core Course
1B03 BMH : LOGIC, SETS AND PROBABILITY THEORY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark.

1. Express the empty set as a subset of Q, the set of rationals.
2. Identify the set $\{x, x \in R : |x| = 4\}$.
3. Show that $E(c) = c$, where c is a constant.
4. The diameter of an electric cable, say X, is assumed to be $f(x) = 6x(1 - x)$, $0 \leq x \leq 1$. Show that f(x) is a p.d.f.
5. Define the variance of a random variable X .

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks.

6. State the Archimedian Property.
7. Define the power set of a set S. What is the power set $P(\phi)$ of the empty set.
8. Find $A \times B$ if $A = \{1\}$ and $B = (-1, 1)$.
9. Define the m.g.f. of a random variable X. Find the m.g.f. of $Y = aX + b$.
10. Find the expectation of the number on a die when thrown.

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11. A coin is tossed four times. Let X denote the number of times a head is followed immediately by a tail. Find the distribution.
12. What is negation of a statement. Write the negation of the statement : $3^2 + 2 = 7$.
13. Let $f : X \to Y$ be a map and $y, z \in Y$. Write the following quantifiers :
    i) y is in the image of f.
    ii) y is not in the image of f.
14. Write the converse and contra positive of the statement: if x is odd, then x^2 is odd.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks **each**.

15. Prove the following by using the method of contradiction: There are infinitely many prime numbers.
16. Describe the set $A = \{x \in R : x(x - 1)(x - 2) < 0\}$ explicitly and mark it on the real line.
17. Suppose $B \subseteq C$. Prove that for any set A
    i) $A \cup B \subseteq A \cup C$
    ii) $A \cap B \subseteq A \cap C$.
18. Given that $A = [-1,1]$ and $B = (0, 1)$. Find
    i) $A \cup B$
    ii) $A \cap B$
    iii) $A \setminus B$
    iv) $B \setminus A$
19. Produce counter examples to disprove the following statements :
    i) For any $x \in R$, $x^2 > x$
    ii) For any $x, y \in R$, $x^2 = y^2$ implies $x = y$
    iii) For any $x, y \in R$, $|x| > |y|$ if $x > y$
    iv) For $x, y \in R$, $x^2 + y^2 > 2xy$.

20. $f(x) = x/15$ when $x = 1, 2, 3, 4, 5$ and 0 elsewhere is the p.d.f. of the random variable X. Find its distribution function and hence find $P(1 \leq X \leq 2)$.
21. The probability mass function of a random variable X is zero except at the points $x = 0, 1, 2$ at these points it has the values $p(0) = 3c^3$, $p(1) = 4c - 10c^2$ and $p(2) = 5c - 1$ for some $c > 0$.
    i) Determine the value of c
    ii) Compute $P(X < 2)$
    iii) Find the largest x such that $F(x) < 1/2$
    iv) Find the smallest x such that $F(x) \geq 1/3$.
22. If a random variable X has the density function $f(x) = \begin{cases} \frac{1}{4}, & -2 < x < 2 \\ 0 & \text{elsewhere} \end{cases}$ obtain
    i) $P(X < 1)$
    ii) $P(|X| > 1)$
    iii) $P(2X + 3 > 5)$.
23. X and Y are independent random variables with $E(X) = 10$, $E(Y) = -5$ and $Var(X) = 4$, $Var(Y) = 16$. Find a and b such that $Z = aX + bY$ will $E(Z) = 0$, $Var(Z) = 28$.
24. Let X be a random variable with p.d.f. $f(x) = 2/3$ when $x = 1$, $1/3$ when $x = 2$ and zero elsewhere. Find the moment generating function and the central moments μ_2 and μ_3.
25. Find the expectation and variance of X if its p.d.f. $f(x) = Ae^{-2x}$, $0 \leq x < \infty$.
26. If X and Y are two independent random variables, prove that m.g.f. of X + Y is equal to the product of m.g.f.'s of X and Y.

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks **each**.

27. If X and Yare two random variables having joint density function $f(x, y) = \frac{1}{8}(6 - x - y)$, $0 < x < 2$, $2 < y < 4$ and 0 otherwise.
    Find
    i) $P(X < 1 \cup Y < 3)$,
    ii) $P(X + Y < 3)$,
    iii) $P(X < 1 | Y < 3)$.
28. The probability density function of a random variable X follows the following probability law : $p(x) = \frac{1}{2\theta}\exp\left(\frac{-|x-\theta|}{\theta}\right), -\infty, x < \infty$. Find M.G.F. of X. Hence find E(X) and Var(X).
29. If X and Y are two random variables, then prove the following :
    i) $E(X + Y) = E(X) + E(Y)$
    ii) If X and Y are independent, $E(XY) = E(X)E(Y)$
    iii) If $X \leq Y$, then $E(X) \leq E(Y)$.
30. State and Prove the De Morgan's Law.
