K22U 0213

Reg. No. : ....

Name : ....

VI Semester B.Sc. Honours in Mathematics Degree (CBCSS-Regular/ Supplementary/Improvement – 2016 Syllabus) Examination, April 2022
BHM 601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries 1 mark. **(4×1=4)**

1. Let $f(t) = e^t$ when $t \geq 0$, a is a constant. Find $\mathcal{L}(f)$.
2. State the convolution theorem for Laplace transforms.
3. Define Fourier sine transform of an odd function f(x).
4. Define Z-transform of a sequence {f(n)} as the function F(z) of a complex variable z.
5. Find $Z^{-1}\{e^{1/z}\}$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. Find the Laplace transform of cosh at.
7. Let $H(s) = 1/[(s - a)s]$. Find inverse transform h(t).
8. Find the Fourier sine transform of the function
$$f(x) = \begin{cases} k, & \text{if } 0 \leq x \leq a \\ 0, & \text{if } x > a \end{cases}$$
9. Find the first order of Hankel transform of $f(r) = e^{-ar}$.
10. Let $f(n) = a^n$, $n \geq 0$. Find $Z\{a^n\}$.
11. Find the inverse Z-transform of $F(z) = z/(z^2 - 6z + 8)$.

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12. Find the Fourier cosine transform of e^{-x}.
13. Find the Mellin transform of the function $f(x) = e^{-nx}$, where $n \geq 0$.
14. Show that $\tilde{f}(k) = \mathcal{H}\left\{\frac{e^{-ar}}{r}\right\} = (1/k)[1 - a(k^2 + a^2)^{-1/2}]$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Solve the initial value problem $y' + (1/2)y = 17 \sin(2t)$, $y(0) = -1$, using the Laplace transform.
16. Find the inverse transform of $F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-2s}}{s^2 + \pi^2} + \frac{e^{-3s}}{(s+2)^2}$.
17. a) Write the properties of convolution.
    b) Let $H(s) = 1/[(s - a)s]$. Find h(t).
18. Represent f(x) as a Fourier cosine integral, $f(x) = \begin{cases} 1, & \text{if } 0 < x < a \\ 0, & \text{if } x > 1 \end{cases}$.
19. State and prove linearity property of Fourier transform.
20. If $f(x) = (e^x - 1)^{-1}$. Find $\mu\{1/(e^x - 1)\}$.
21. Find the first order Hankel transform of
    a) $f(r) = \sin ar/r$
    b) $f(r) = r.e^{(-ar^2)}$.
22. If $\mathcal{H}_n\{f(r)\} = \tilde{f}_n(k)$, then show that $\mathcal{H}_n\{f(ar)\} = \frac{1}{a^2}\tilde{f}_n\left(\frac{k}{a}\right)$, $a > 0$.
23. Show that, $\mu\left\{\frac{1}{(1+x)^n}\right\} = \frac{\Gamma(p)\Gamma(n-p)}{\Gamma(n)}$.
24. Show that $Z\{n^2\} = z(z + 1)/(z - 1)^3$.
25. Find the inverse Z-transform of $F(z) = \frac{3z^2 - z}{(z-1)(z-2)^2}$.
26. Find the sum of the series $\sum_{n=0}^{\infty} a^n \sin nx$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. a) Find the Laplace transform of sinh at.
    b) Let f(x) be continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$. Furthermore, let f′(x) be absolutely integrable on the x-axis. Then show that $\mathcal{F}(f'(x)) = i\omega\mathcal{F}\{f(x)\}$.
28. State and prove second shifting theorem for Laplace transforms.
29. State and prove the final value theorem for Z-transforms.
30. a) Use the convolution theorem to show that $Z^{-1}\left\{\frac{z(z+1)}{(z-1)^3}\right\} = n^2$.
    b) Solve the initial value problem for the difference equation $f(n + 1) - f(n) = 1$, $f(0) = 0$.

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