Reg. No. : ....

Name : ....

K22U 0214

VI Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Regular/ Supplementary/Improvement – 2016 Syllabus) Examination, April 2022

BHM 602 : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

(Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.) **(4×1=4)**

1. Write the discrete topology on a set S = {1, 2, 3}.
2. Define the usual topology on the set of real numbers.
3. Give an example for a set which is both open and closed in a topological space.
4. Define embedding of a topological space into another.
5. Define normal space and write an example for such a space.

SECTION – B

(Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.) **(6×2=12)**

6. Prove that the semi-open interval topology is stronger than the usual topology on the set of real numbers.
7. Prove that the product topology is the weak topology determined by the projection function.
8. If A and B are any subset of a topological space X, prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.
9. Let $Z \subset Y \subset X$, and be a topology on X. Then prove that $(\tau/Y)/Z = \tau/Z$.
10. Prove that every T_2 space is T_1.

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11. Suppose y is an accumulation point of a subset A of a T_1 space X. Then prove that every neighborhood of y contains infinitely many points of A.
12. Prove that regularity is a hereditary property.
13. Prove that every compact Hausdorff space is a T_3 space.
14. Prove that every regular, second countable space is normal.

SECTION – C

(Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.) **(8×4=32)**

15. If a space is second countable then prove every open cover of it has a countable subcover.
16. Prove that metrisability is a hereditary property.
17. Prove that for a subset A of a space X, $\overline{A} = A \cup A'$.
18. Prove that the interior of a set is the same as the complement of the closure of the complement of the set.
19. Let (X, τ) be a topological space and $A \subset X$. Then prove that A is a compact subset of X if and only if the subspace (A : J/A) is compact.
20. Prove that every continuous real-valued function on a compact space is bounded and attains its extrema.
21. Prove that every separable space satisfy the countable chain condition.
22. Prove that all metric spaces are T_4.
23. Prove that every completely regular space is regular and also prove every Tychnoff space is T_3.
24. Prove that every regular, Lindeloff space is normal.
25. Prove that second countable space is also first countable.
26. Let X be completely regular space. Suppose F is a compact subset of X, C is a closed subset of X and $F \cap C = \phi$. Then prove that there exists a continuous function from X into the unit interval which takes the value 0 at all points of F and the value 1 at all points of C.

SECTION – D

(Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.) **(2×6=12)**

27. Prove that closed subset of a compact space is compact.
28. Prove that every closed and bounded interval is compact.
29. Prove that :
    a) Components are closed sets.
    b) Any two distinct components are mutually disjoint.
    c) Every nonempty connected subset is contained in a unique component.
    d) Every space is the disjoint union of its components.
30. Let X, Y be spaces, $x \in X$ and $f : X \to Y$ a function. Suppose X is first countable at x. Then prove that f is continuous at x if and only if for every sequence $\{x_n\}$ which converges to $x \in X$, the sequence $\{f(x_n)\}$ converges to f(x) in Y.
