Reg. No. : ....

Name : ....

K22U 2602

V Semester B.Sc. Hon's (Mathematics) Degree (CBCSS – Regular/ Supplementary/Improvement) Examination, November 2022
(2017 Admission Onwards)
BHM 502 : ADVANCED COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. What do you mean by a contour ?
2. Define singular point.
3. Find $\lim_{n \to \infty} z_n$ where $z_n = \frac{1 + in^3}{n^3}$.
4. Find the residue at z = 0 of $f(z) = \frac{z - \sin z}{z}$.
5. Write the series expansion of $z^2 \sin\left(\frac{1}{z}\right)$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Prove that if a series converges its n^{th} term converges to zero.
7. Evaluate $\int_C \bar{z}\, dz$ where curve C is the line from z = 0 to z = 2i.
8. Find the value of the integral of $g(z) = \frac{1}{z^2 + 4}$ around the circle $|z - i| = 2$ in the positive sense.
9. Find all the singular points of the function $f(z) = \frac{1}{\sin\left(\frac{\pi}{z}\right)}$ and classify whether they are isolated or not.

P.T.O.

10. State Cauchy's Residue Theorem.
11. Let C denote the unit circle |z| = 1 described in positive sense. Determine the value of $\Delta_C \arg f(z)$ when $f(z) = z^2$.
12. Find the residue of the function $f(z) = \frac{\tanh z}{z^2}$ at $z = \frac{\pi i}{2}$.
13. Define circle of convergence.
14. Evaluate $\int_0^{\pi/4} e^{it} dt$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. If w(t) is a piecewise continuous complex valued function defined on an interval $a \leq t \leq b$, then prove that $\left|\int_a^b w(t)dt\right| \leq \int_a^b |w(t)|dt$.
16. Let C be the arc of the circle |z| = 2 from z = 2 to z = 2i that lies in the first quadrant. Show that $\left|\int_C \frac{z+4}{z^3 - 1} dz\right| \leq \frac{6\pi}{7}$ on the arc C.
17. State and prove Liouville's theorem.
18. Evaluate the integral $\int_0^\infty \frac{x^2}{x^6 + 1} dx$.
19. Determine the number of roots of the equation $z^7 - 4z^3 + z - 1 = 0$ inside the circle |z| = 1.
20. Prove that $\sum_{n=1}^\infty \frac{z^n}{n(n+1)}$ converges absolutely for $|z| \leq 1$.
21. Find Laurent series at the indicated singularity and name the singularity for the function $f(z) = \frac{z}{(z+1)(z+2)}$ at z = –2.

22. Prove that a power series $\sum_{n=0}^\infty a_n (z - z_0)^n$ represents a continuous function S(z) at each point inside its circle of convergence $|z - z_0| = R$.
23. Find the pole, its order and residue of the function $f(z) = \frac{\sinh z}{z^4}$.
24. Show that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1 - a^2}}\ (-1 < a < 1)$.
25. Suppose that z_0 is an essential singularity of a function f and let w_0 be any complex number then prove that for any positive number ε, the inequality $|f(z) - w_0| < \varepsilon$ is satisfied at some point z in each deleted neighbourhood $0 < |z - z_0| < \delta$ of z_0.
26. State and prove Cauchy integral formula.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. State and prove Fundamental Theorem of Algebra.
28. State and prove maximum modulus principle.
29. Show that $\int_0^\infty \frac{\ln x}{(x^2 + 4)^2} dx = \frac{\pi}{32}(\ln 2 - 1)$.
30. Prove the statement "Let a function f be analytic at a point z_0. It has a zero of order m at z_0 iff there is a function g, which is analytic and nonzero at z_0 such that $f(z) = (z - z_0)^m g(z)$.