K22U 2603

Reg. No.: ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (CBCSS – Regular/ Supplementary/Improvement) Examination, November 2022
(2017 Admission Onwards)
BHM503 : ADVANCED DISCRETE MATHEMATICS

Time : 3 Hours | Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define Eulerian graph.
2. State Hall's condition for a bipartite graph.
3. How many derangements are there for 1, 2, 3, 4, ?
4. Define the generating function for a given sequence of real numbers $a_0, a_1, a_2,$ ________
5. Find the exponential generating function for the sequence $1, 2, 2^2, 2^3,$ ________

(4×1=4)

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Construct the de Bruijn digraph B(3, 2).
7. Let G be a graph of order $n \geq 2$. Show that if $\deg u + \deg v \geq n - 1$ for each pair u, v of non adjacent vertices of G then G contains a Hamiltonian path.
8. Define matching of a graph and give an example for matching.
9. Show that for each positive integer k, the complete graph K_{2k} is1-factorable.

10. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, or pun occurs ?
11. Compute ϕ (n) for n = 2100.
12. Find the coefficient of x^5 in $(1 - 2x)^{-7}$.
13. Verify that for all $n \in \mathbb{Z}^+$, $\binom{2n}{n} = \sum_{i=0}^{n}\binom{n}{i}^2$.
14. Obtain the convolution of the sequences 1, 1, 1, 1, ... and 1, –1, 1, –1, ... using their generating functions.

(6×2=12)

SECTION – C

Answer **any 8** questions out of 12 questions . **Each** question carries **4** marks.

15. Prove that the Petersen graph is not Hamiltonian.
16. Define Hamiltonian connected graph. Prove that no bipartite graph of order 3 or more is Hamiltonian connected.
17. Define perfect matching. Prove that every r-regular bipartite graph $(r \geq 1)$ has a perfect matching.
18. Explain the concept of clique and clique number of a graph with suitable example.
19. Explain the concept of k – factor and factorization of a graph. Give an example of a cubic graph with a 1 – factor that is not 1 – factorable.
20. Show that for every positive integer k, the complete graph K_{2k+1} is Hamiltonian factorable.
21. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5.
22. Let A = {1, 2, 3, ... , 7}. How many one to one functions $f : A \to A$ have at least one fixed point ?
23. Explain the concept of rook polynomials with a suitable example.

24. a) Find the generating function for the number of ways to select 10 candy bars from large supplies of six different kinds.

    b) Find the generating function for the number of ways to select , with repetitions allowed, r objects from a collection of n distinct objects.
25. Use generating functions to determine how many four element subsets of S = {1, 2, 3, ...,15} contain no consecutive integers ?
26. Determine the sequence generated by each of the following exponential generating functions :

    a) $f(x) = e^{2x} - 3x^3 + 5x^2 + 7x$

    b) $f(x) = \frac{3}{(1-2x)}$.

(8×4=32)

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Let G be a graph of order $n \geq 4$. Prove that if $\deg u + \deg v \geq n + 1$ for each pair u, v of non adjacent vertices of G then G is Hamiltonian connected.
28. Prove that for every connected graph G, $\alpha(G) \geq \text{rad } G$.
29. Find the number of integer solutions for the following equations.

    i) $c_1 + c_2 + c_3 + c_4 + c_5 = 30$, $2 \leq c_1 \leq 4$ and $3 \leq c_i \leq 8$ for all $2 \leq i \leq 5$.

    ii) $c_1 + c_2 + c_3 + c_4 + c_5 = 30$, $0 \leq c_i$ for all $1 \leq i \leq 5$ with c_2 even and c_3 odd.
30. Find the coefficient of x^{13} in each of the following :

    a) $x^3(1 - 2x)^{10}$

    b) $\frac{(x^3 - 5x)}{(1-x)^3}$

    c) $\frac{(1+x)^4}{(1-x)^4}$.

(2×6=12)