K22U 3676

Reg. No. : ....

Name : ....

Third Semester B.Sc. Honours in Mathematics Degree (CBCSS – Supplementary/Improvement) Examination, November 2022 (2017 – 2020 Admissions) BHM 301 : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. State Bernoulli's inequality.
2. Give an example of a divergent sequence (x_n) of positive numbers with $\lim \left(x_n^{1/n}\right) = 1$.
3. Define a step function.
4. Define rearrangement of a series.
5. State Supremum Property of R.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Write example of a set having supremum, but the supremum does not belong to the set.
7. Check whether the nested sequence (0, 1/n), $n \in N$ has a common point.
8. Give an example of two divergent sequences X and Y such that XY converges.
9. Discuss the convergence of (1, 1/2, 3, 1/4, ...).
10. State and prove the n^{th} term test of the series.
11. Show that if a series in R is absolutely convergent, then it is convergent.
12. Show by an example that continuous functions need not be bounded.

P.T.O.

13. Every uniformly continuous functions are Lipschitz function. True/False. Justify.
14. Write two uniformly continuous function on R such that their product is not uniformly continuous function on R.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Show that the unit interval [0, 1] is not countable.
16. State and prove Archimedean property.
17. Show that $-|a| \le a \le |a|$ for all $a \in R$.
18. State Squeeze Theorem. Hence find $\lim\left(\frac{\sin n}{n}\right)$.
19. Show that Cauchy sequence is convergent.
20. Let $x_1 = 8$ and $x_{n+1} = \frac{1}{2}x_n + 2n \in N$. Show that (x_n) is bounded and monotone. Find the limit.
21. State the Integral Test. Discuss the convergence of p-series using it.
22. State and prove Abel's Test.
23. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{1}{(n^2 - n + 1)}$.
24. State and prove Bolzano's Intermediate Value Theorem.
25. Show that we can approximate continuous functions by continuous piecewise linear functions.
26. If δ is a gauge defined on the interval [a, b], then show that there exist a δ-fine partition of [a, b].

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. State and prove Monotone Convergence Theorem.
28. State and prove Uniform Continuity Theorem.
29. Show that the set of rational numbers are dense in R.
30. Discuss the convergence of alternating harmonic series.