Reg. No. : ....

Name : ....

Third Semester B.Sc. Honours in Mathematics Degree (CBCSS – Regular) Examination, November 2022
(2021 Admission)
3B09BMH : REAL ANALYSIS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. State principle of strong induction.
2. State Bernoulli's inequality.
3. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
4. Give an example of a convergent sequence (x_n) of positive terms with $\lim\left(x_n^{\frac{1}{n}}\right) = 1$.
5. If the series $\sum x_n$ converges, prove that $\lim x_n = 0$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. State and prove Cantor's theorem.
7. If $a, b \in \mathbb{R}$, prove that $||a| - |b|| \le |a - b|$.
8. Determine the set $B = \{x \in \mathbb{R} : |x - 1| < |x|\}$.
9. State and prove Archimedean property.

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10. Prove that a sequence in $\mathbb{R}$ can have at most one limit.
11. Prove that a convergent sequence of real numbers is bounded.
12. State and prove squeeze theorem.
13. State and prove rearrangement theorem.
14. State and prove Abel's test.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Prove that $n^3 + 5n$ is divisible by 6.
16. Prove that the set $\mathbb{N} \times \mathbb{N}$ is denumerable.
17. Let S be a non-empty subset of $\mathbb{R}$ that is bounded above, and let a is any real number. Prove that $\sup(a + S) = a + \sup S$.
18. State and prove nested interval property.
19. Prove that the set $\mathbb{R}$ of real numbers is not countable.
20. Prove that $\lim n^{\frac{1}{n}} = 1$.
21. State divergence criteria and using this criteria prove that $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \dots\right)$ is divergent.
22. Let (x_n) be defined by $x_1 = 1$, $x_2 = 2$ and $x_n = \frac{1}{2}(x_{n-2} + x_{n-1})$ for $n > 2$. Prove that (x_n) is a Cauchy sequence.
23. Prove that $\sum_{n=1}^{\infty} r^n$ is convergent if $|r| < 1$ and divergent if $|r| \ge 1$.
24. Prove that $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)} = 1$.
25. Does the series $\sum_{n=1}^{\infty}\left(\frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n}}\right)$ converges ?
26. State and prove root test.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Prove that the following statements are equivalent.
    a) S is a countable set
    b) There exists a surjection of $\mathbb{N}$ onto S
    c) There exists an injection of S into $\mathbb{N}$.
28. Let S be a subset of $\mathbb{R}$ that contains at least two points and has the property if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$. Prove that S is an interval.
29. State and prove monotone convergence theorem.
30. a) State and prove interval test.
    b) Discuss the convergence or divergence of the series $\sum \frac{1}{n \ln n}$.