K22U 3672

Reg. No. : ....

Name : ....

Third Semester B.Sc. Honours in Mathematics Degree (CBCSS – Regular) Examination, November 2022
(2021 Admission)
3B10 BMH : CALCULUS – III

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any four** questions out of **five** questions. **Each** question carries **one** mark.

1. State Fubini's theorem for double integrals.
2. Find the cylindrical point of the point with rectangular co-ordinates (3, – 3, –7).
3. Find the Jacobian of $x = u^2 - v^2$, $y = 2uv$.
4. Find the gradient of the vector field $f(x, y, z) = x^3y\, i + y^4z\, j + xyz\, k$.
5. Find the Curl F, where $F = xy\, i + xz\, j + yz\, k$.

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. Evaluate the integral $\iint_R \sqrt{1-x^2}\, dx$, $R = \{(x, y) : -1 \le x \le 1, -2 \le y \le 2\}$.
7. Find $\int_1^2\int_0^3 x^2y\, dx\, dy$.
8. Find the mass of a triangular lamina with vertices (0, 0), (1, 0) and (0, 2).
9. Evaluate $\iiint_B xyz^2\, dV$, where V is the rectangular box given by $B = \{(x, y, z)/0 \le x \le 1, -1 \le y \le 2, 0 \le z \le 3\}$.

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10. Use triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$.
11. Sketch the vector field on R^3, given by $F(x, y, z) = zk$.
12. Find the work done by the force field $F(x, y) = x^2 i - xy\, j$, in a moving particle along the quarter – circle $r(t) = \cos t\, i + \sin t\, j$, $0 \le t \le \frac{\pi}{2}$.
13. Show that the vector field $F(x, y) = (3 + 2xy)i + (x^2 - 3y^2)j$ is conservative.
14. Find the flux of the vector field $F(x, y, z) = xi + zk$, over $x^2 + y^2 + z^2 = a^2$.

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy plane and inside the cylinder $x^2 + y^2 = 2x$.
16. Evaluate the iterated integral $\int_0^1\int_x^1 \sin(y^2)\, dy\, dx$.
17. Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)\, dz\, dy\, dx$.
18. Evaluate $\iiint_B e^{(x^2+y^2+z^2)^{\frac{3}{2}}}\, dV$, where $B = \{(x, y, z) : x^2 + y^2 + z^2 \le 1\}$.
19. Evaluate $\int_C (2 + x^2y)ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$.
20. Find the potential function of the vector field $F(x, y, z) = y^2i + (2xy + e^{3z})j + 3ye^{3z}\, k$.
21. Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
22. Using Green's theorem, evaluate $\int_C x^4dx + xydy$, where C is a triangular curve consisting of the line segments from (0, 0) to (1, 0) from (1, 0) to (0, 1) and from (0, 1) to (0, 0).
23. Find the flux of the vector field $F(x, y, z) = z\, i + y\, j + x\, k$, across the unit sphere $x^2 + y^2 + z^2 = 1$.

24. Evaluate $\int_C F.dr$, where $F(x, y, z) = -y^2\, i + x\, j + z^2\, k$, where C is the curve of intersection of the plane $y + z = 2$ and the cylinder $x^2 + y^2 = 1$.
25. The temperature u in a metal ball is proportional to the square of the distance from the center of the ball. Find the rate of heat flow across a sphere S of radius a with center at the center of the ball.
26. Evaluate $\iint_S y\, dS$, where S is the surface $z = x^2 + y^2$, $0 \le x \le 1$, $0 \le y \le 2$.

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. The density at any point on a circular lamina is proportional to the distance from the center of the circle. Find the center of mass of lamina.
28. Evaluate $\iiint_E \sqrt{x^2 + z^2}\, dV$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$.
29. If $F(x, y) = -\frac{y}{x^2 + y^2}i + \frac{x}{x^2 + y^2}j$, show that $\int_C F.dr = 2\pi$ for every positively oriented simple closed path that encloses the origin.
30. Evaluate $\iint_S F.dS$, where $F(x, y, z) = xy\, i + (y^2 + e^{xz^2})\, j + \sin(xy)\, k$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$ and the plane $z = 0$, $y = 0$ and $y + z = 2$.

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