Reg. No. : ....

Name : ....

Third Semester B.Sc. Honours in Mathematics Degree (CBCSS – Supplementary/Improvement) Examination, November 2022 (2017-2020 Admissions)
BHM 302 : VECTOR CALCULUS

Time : 3 Hours Max. Marks : 60

Answer **any 4** out of 5 questions. **Each** question carries **1** mark :

1. Define limit of a vector function.
2. Define length of a smooth curve.
3. State Fubini's theorem.
4. State Fundamental theorem of line integrals.
5. Define Stoke's theorem.

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks :

6. If $\vec{r}(t) = \cos t i + \sin t j + tk$, then find $\lim_{t \to \frac{\pi}{4}} \vec{r}(t)$.
7. Find the velocity and acceleration of a particle whose motion in space is given by the position vector $\vec{r}(t) = 2\cos t i + 2\sin t j + 5\cos^2 t k$.
8. A straight line is parametrized by $\vec{r}(t) = \vec{C} + t\vec{v}$ for constant vector $\vec{C}$ and $\vec{v}$. Find its curvature.
9. Calculate $\iint_R f(x,y)\,dA$ for $f(x, y) = 100 - 6x^2y$ and $R : 0 \le x \le 2, -1 \le y \le 1$.
10. Find the volume of the region bounded above by the elliptical paraboloid $z = 10 + x^2 + 3y^2$ and below by the rectangle $R : 0 \le x \le 1, 0 \le y \le 2$.
11. Evaluate $\iint_R e^{(x^2+y^2)}\,dydx$ where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1-x^2}$.

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12. Show that $\vec{F} = (2x-3)i - zj + \cos zk$ is not conservative.
13. Find the workdone by the conservative field $F = \nabla f$ where $f = xyz$ along any smooth curve C joining the point A (– 1, 3, 9) to B (1, 6, – 4).
14. Prove that (curlgrad) f = 0.

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks :

15. Find the arclength parametrization of a helix $\vec{r}(t) = \cos t i + \sin t j + tk$ from t_0 to t.
16. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of $\vec{v} = 3i - 4j$.
17. Find the principal unit normal vector N for the helix $\vec{r}(t) = a\cos t i + a\sin t j + btk$, $a, b \ge 0$, $a^2 + b^2 \ne 0$ and describe how the vector is pointing.
18. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point P_0 (1, 2, 4).
19. Find the volume of the solid region bounded above by the paraboloid $z = 9 - x^2 - y^2$ and below by the unit circle in the xy-plane.
20. Find the average value of $F(x, y, z) = xyz$ throughout the cubical region D bounded by the coordinate planes and the planes x = 2, y = 2, z = 2 in the first octant.
21. Find the centroid ($\delta = 1$) of the solid enclosed by the cylinder $x^2 + y^2 = 4$ bounded above by the paraboloid $z = x^2 + y^2$ and below by the xy-plane.
22. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).
23. Evaluate $\int_C \vec{F}.d\vec{r}$ where $\vec{F}(x,y,z) = zi + xyj - y^2k$ along the curve C given by $\vec{r}(t) = t^2 i + tj + \sqrt{t}k, 0 \le t \le 1$.
24. A fluid's velocity field is $\vec{F} = xi + zj + yk$. Find the flow along the helix $\vec{r}(t) = \cos t i + \sin t j + tk, 0 \le t \le \frac{\pi}{2}$.

25. Find the surface area of the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.
26. Verify Divergence theorem for the vector field $\vec{F} = xi + yj + zk$ over the sphere $x^2 + y^2 + z^2 = a^2$.

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks :

27. Find the principal unit normal vector N and unit tangent vector T for the circular motion $\vec{r}(t) = \cos 2ti + \sin 2tj$.
28. Integrate F (x, y, z) = 1 over the tetrahedron D with vertices (0, 0, 0), (1, 1, 0), (0, 1, 0) and (0, 1, 1) in the order dzdydx.
29. Show that $\vec{F} = (e^x \cos y + yz)i + (xz - e^x \sin y)j + (xy + z)k$ is conservative over its natural domain and find a potential function for it.
30. Find the surface area of a sphere of radius a.