Reg. No. : ....

Name : ....

Third Semester B.Sc. Honours in Mathematics Degree (C.B.C.S.S. – Regular) Examination, November 2022 (2021 Admission) 3B11 BMH : GRAPH THEORY

Time : 3 Hours Max. Marks : 60

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Define regular graph.
2. Give an example of a graph in which the length of the longest cycle is 9 and the length of the shortest cycle is 4.
3. If the connected graph G has 17 edges, what is the maximum possible number of vertices in G ?
4. Which of the complete graphs K_n, for $n \geq 3$ are Euler ?
5. Give an example of a 4-critical graph.

PART – B

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

6. Define graph isomorphism. Explain with example.
7. Prove that in any graph G there is an even number of odd vertices.
8. Prove that three is no simple graph with six vertices, one of which has degree 2, two have degree 3, three have degree 4 and the remaining vertex has degree 5.
9. State any two characterization of trees.
10. Let G be a connected graph with at least three vertices. Prove that if G has a bridge the G has a cut vertex.

P.T.O.

11. State the marriage problem.
12. Prove that the wheel graph W_n is Hamiltonian for every $n \geq 4$.
13. Define Jordan curve. State Jordan curve theorem.
14. Let G be a k-critical graph. Then prove that G is connected.

PART – C

Answer **any 8** questions from this Part. **Each** question carries **4** marks. **(8×4=32)**

15. State a real life situation and its graph model.
16. Explain the Matrix representation of graphs using incidence matrices with example.
17. Let G be a graph with n vertices $v_1, v_2, \ldots, v_n$ and let A denote the adjacency matrix of G with respect to this listing of vertices. Let k be any positive integer and let A^k denote the matrix multiplication of k copies of A. Then the $(i, j)^{th}$ entry of A^k is the number of different $v_i - v_j$ walks in G of length k.
18. a) Let u and v be distinct vertices of a tree T. Then prove that there is precisely one path from u to v.

    b) Let G graph without any loops. If for every pair of distinct vertices u and v of G there is precisely one path from u to v, then prove that G is a tree.
19. Prove that an edge e of a graph G is a bridge if and only if e is not part of any cycle in G.
20. Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has at least two vertices which are not cut vertices.
21. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.
22. Let G be a simple graph on n vertices, with $n \geq 3$. If c(G) is complete, then prove that G is Hamiltonian.
23. Define Hamiltonian graphs. Let G be a simple graph with n vertices and let u and v be non-adjacent vertices in G such that $d(u) + d(v) \geq n$. Then prove that G is Hamiltonian if and only if G + uv is Hamiltonian.

24. Let G be a simple 3-connected graph with at least five vertices. Then prove that G has a contractible edge.
25. Let G be a nonempty graph. Then prove that $\chi(G) = 2$ if and only if G is bipartite.
26. Let G be a graph with $\chi(G) = k$. Then prove that G has at least k vertices v such that $d(v) \geq k - 1$.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

27. Let G be a nonempty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.
28. State and prove the Whitney's theorem on 2-connected graphs.
29. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
30. Prove that K_5 is nonplanar.

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