K22U 1317

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – Supplementary/Improvement) Examination, April 2022
(2016 – 2020 Admissions)
BHM 202 : ABSTRACT ALGEBRA AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Whether * defined as $a * b = c$ where c is at least 5 more than b, is a binary operation on the set of positive integers Z^+, justify your answer.
2. Define cyclic group.
3. Give an example of a finite group that is not cyclic.
4. Define subspace of a vector space.
5. Check whether $T : R^3 \to R^3$ defined by $T(x, y, z) = (x - 1, y, z)$ is a linear map. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Check whether the binary operation * defined by letting $a * b = ab + 1$ on Q is commutative and associative.
7. Is the set {1, 2, 3, 4, 5, 6, 7} a group under multiplication modulo 8 ?
8. Find the order of the subgroup of Z_4 (with addition modulo 4) generated by 3.
9. Find the product of the cycles (1, 3, 6) and (2, 6) in S_6.
10. Determine whether the set $\{1 + x, x + x^2, x^2 + 1\}$ of polynomials of degree ≤ 2 is linearly independent or dependent.
11. Define vector space with example.

P.T.O.

12. Define basis of a vector space. Give a basis for space of polynomial of degree ≤ 3.
13. Define linear transformation. Give an example.
14. Define isomorphism in linear transformation. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Show that a set $A = \{1, \omega, \omega^2\}$ where ω is a cube root of unity, forms an abelian group with respect to multiplication.
16. Show that the set of all n × n non-singular matrices having their elements as rational numbers is a non-abelian group with matrix multiplication as composition.
17. Show that the identity element and inverse of each element in a group is unique.
18. Show that $a^m = e$, if and only if n divides m where a is the generator of a cyclic group G of order n.
19. If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$, compute σ^{100}.
20. Find the order of the permutations $\tau = (1, 4)\ (3, 5, 7, 8)$.
21. Show that if U and W are 2 subspaces of a vector space V, show that $U \cap W$ is also a subspace of V.
22. If $U = \{P(x) \in P_3 : P''(2) = 0\}$, find a basis for U and hence find dim U.
23. If {u, v, w} is linear independent in a vector space V, check whether {u – v, v – w, v –u} is linearly independent.
24. Let T be a linear transformation from V into W where V and W are vector spaces over the field F. If T is invertible, then show that the inverse function T^{-1} is a linear transformation from W into V.
25. Define null space of a transformation. Show that null space of the transformation is a subspace.
26. Define the matrix of a linear transformation. Write the matrix of transformation corresponding to the space of polynomials of degree less than or equal to 3 with the differentiation operator D. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Show that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H$, for all a, b$\in$H.
28. Define permutation of a set. Prove that S_A, the collection of all permutations of A, a non-empty set is a group under permutation multiplication.
29. If S is a non-empty subset of a vector space V, then show that span S is a subspace of V and <S> = Span S.
30. If $T : V \to W$ is a linear transformation where V and W vector spaces over the field F and V is finite dimensional, then show that rank (T) + nullity (T) = dim V. **(2×6=12)**