K22U 1319

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – Supplementary/Improvement) Examination, April 2022
(2016-2020 Admissions)
BHM 204 : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. When two numbers a and b are said to be relatively prime ?
2. State Chinese remainder theorem.
3. Find the sum of divisors of 180.
4. Form an equation whose roots are 3 times those of equation $2x^3 - 5x^2 + 7 = 0$.
5. Write Descarte's Rule of signs. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks :

6. Define a pseudo prime. Illustrate with examples.
7. Use Euclidean algorithm to find gcd(56, 72).
8. Show that $n^7 - n$ is divisible by 42.
9. Find the remainder when 2^{1000} is divided by 17.
10. Check whether the Diophantine equation $6x + 51y = 22$ can be solved.
11. If gcd(a,b) = 1 then prove that gcd(a + b, a – b) = 1 or 2.

P.T.O.

12. Find the polynomial equation of the lowest degree with rational coefficients having $\sqrt{3}$ and $1 - 2i$ as two of its roots.
13. Form a rational quartic whose roots are 1, –1, $2 + \sqrt{3}$.
14. Remove the second term from the equation $x^3 - 6x^2 + 4x - 7 = 0$. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks :

15. Find the remainder when 18! is divided by 23.
16. Use mathematical induction to show that 8 divides $5^{2n} + 7$, for $n \geq 1$.
17. Use Euclidean algorithm to obtain integers x and y such that gcd(119, 272) = 119x + 272y.
18. Solve the linear congruence $143x \equiv 47 \pmod{20}$.
19. Determine all solutions of the Diophantine equation $54x + 21y = 906$.
20. State and prove Little Fermat's Theorem.
21. Use division Algorithm to show that cube of any integer is of the form 7k or $7k \pm 1$.
22. Solve the linear system $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 4$, and $x \equiv 3 \pmod 5$.
23. Show that the equation $x^5 - 6x^2 - 4x + 5 = 0$ has atleast two imaginary roots.
24. Solve the equation $x^3 - 12x - 65 = 0$ using Cardan's method.
25. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - 2x^3 + 2x^2 + 1 = 0$, form the equation whose roots are $2+\frac{1}{\alpha}$, $2+\frac{1}{\beta}$, $2+\frac{1}{\gamma}$, $2+\frac{1}{\delta}$ and hence evaluate

    $(2\alpha + 1)(2\beta + 1)(2\gamma + 1)(2\delta + 1)$.
26. If a, b, c, d are the roots of the equation $x^4 - px^3 - rx + s = 0$, find the condition that ab = cd. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks :

27. State and prove Wilson's theorem.
28. State and prove division algorithm theorem.
29. If α, β, γ be the roots of the equation $ax^3 + bx^2 + cx + d = 0$, find the values of

    i) $\sum(\alpha+\beta)^2$

    ii) $\sum\frac{1}{\alpha\beta}$

    iii) $\sum\frac{\alpha}{\beta\gamma}$
30. Solve the equation $x^6 - 9x^5 + 21x^4 - 21x^2 + 9x - 1 = 0$. **(2×6=12)**

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