K22U 1318

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (C.B.C.S.S. – Supplementary/ Improvement) Examination, April 2022
(2016 – 2020 Admissions)
BHM 203 : INTEGRAL CALCULUS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Evaluate $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$.
2. The volume generated by revolving about the x axis an area bounded by y = f(x) and two ordinates x = a and x = b is given by
3. Sum the series $\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+....$
4. $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$ equals
5. Find the limit of the sequence $a_n = \frac{n^2-2n}{3n^2+n}$.

SECTION – B

Answer **any 6** questions. **Each** question carries **2** marks. **(6×2=12)**

6. Discuss the convergence of the series $\Sigma \frac{n^2}{3^n}$.
7. If $I_n = \int_0^{\frac{\pi}{3}} \tan^n x\, dx$, show that $(n-1)(I_n + I_{n-2}) = \sqrt{3}^{\,n-1}$.
8. Find the area between the curves y = 2 sinx and y = sin2x for $0 \leq x \leq \pi$.
9. Find the mean value of $f(x) = 3x^2 - 3$ on [0, 1]. Does f actually take on this value at some point in the given domain ?

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10. Find the length of the curve y = log secx between the points x = 0 and $x = \frac{\pi}{3}$.
11. Show that every convergent sequence is bounded.
12. Find the sum of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...+\frac{1}{n(n+1)}+...$
13. Evaluate $\int_0^1 x^{\frac{3}{2}}(1-x)^{\frac{3}{2}}dx$.
14. Find the sum $\sum_{n=1}^{\infty} \frac{2^n-1}{3^n}$.

SECTION – C

Answer **any 8** questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the Maclaurin series expansion of log (1 + x).
16. Find the area bounded by the curve $xy^2 = 4a^2(2a - x)$ and its asymptote.
17. Let $(x) = x^3$, $0 \leq x \leq 1$. Then prove that f is Riemann integrable over [0, 1].
18. Find the moments, mass and centre of mass of a triangular plate with vertices at (0, 0), (1, 0) and (1, 2) and which has a constant density $\delta = 3g/cm^2$.
19. Show that the series $\Sigma \frac{1}{n!}$ is convergent.
20. Find the area of the region enclosed by the parabola $x = y^2$ and the line x = y + 2.
21. Find the length of the curve $y = x^{\frac{3}{2}}$ from x = 0 to x = 4.
22. If $\phi(n) = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, show that $\phi(n) + \phi(n-2) = \frac{1}{n-1}$. Hence find $\phi(4)$.
23. Find the area enclosed by one arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ and its base.
24. If $(x) = \frac{1}{1-x}$, write the series expansions of f′(x) and f″(x).
25. Find first four terms in the Maclaurin series expansion of $\sin^{-1}x$.
26. Show that the series $\Sigma \frac{1}{n}$ diverges.

SECTION – D

Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

27. Find the surface area of the solid generated by the revolution of the lemniscate $r^2 = a^2 \cos 2\theta$.
28. Discuss the convergence of the power series $1-\frac{x-3}{3}+\frac{(x-3)^2}{3^2}-\frac{(x-3)^3}{3^3}+...$
    Also find the radius of convergence of the power series.
29. Obtain the reduction formula for $\int \sin^p x \cos^q x\, dx$. Hence evaluate $\int \sin^3 x \cos^2 x\, dx$.
30. The solid lies between the planes perpendicular to the x axis at x = 0 and x = 4. The cross sections perpendicular to the axis on the interval $0 \leq x \leq 4$ are squares whose diagonals run from the parabola $y = -\sqrt{x}$ to the parabola $= \sqrt{x}$. Find the volume of the solid.

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