I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – Supplementary/
Improvement) Examination, November 2022
(2016 – 2020 Admissions)
BHM 102 : FOUNDATIONS OF MATHEMATICS

Time : 3 Hours

Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define partition of a set.

2. Define countable set.

3. Define sphere.

4. In cylindrical co-ordinates the equation r = 4 represents _____.

5. Find the range of the function f : [–2, 2] →[– 4, 4] defined by f(x) = x².

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. If f : A→B, g : B→C are injective maps, then show that g∘f : A→C is injective.

7. Is subset of a countable set countable ? Justify your answer.

8. Write the meaning of the statement (∃x)(∃y) [x + y = 0], where x and y are numbers.

9. Find a spherical coordinate equation for the sphere x² + y² + (z – 1)² = 1.

10. Find the radius and centre of the sphere x² + y² + z² – 2x + 4y – 6z = 2.

11. Prove "Every section of a right circular cone by a plane perpendicular to its axis is a circle".

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12. Define a cone.

13. The normal at any point P of a central conicoid meets the three principal planes at G_1, G_2, G_3, show that PG_1, PG_2, PG_3 are in a constant ratio.

14. Define enveloping cylinder of the quadric.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Define equivalence class and prove that any two equivalence classes are either disjoint or equal.

16. Prove that "A finite product of countable sets is countable".

17. Let f: Z→Z defined as f(x) = x + 1. Is f bijective ? Justify.

18. Write the contrapositive, converse of the conditional statement "The home team wins whenever it is raining".

19. Prove that there are infinitely many prime numbers.

20. Prove that any diametral plane of a paraboloid cuts it in a parabola and that parallel diametral planes cut it in equal parabolas.

21. Find the equations to the tangent planes to 7x² – 3y² – z² + 21 = 0, which pass through the line 7x – 6y + 9 = 0, z = 3.

22. Find the equations of the cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is the ellipse x² + 2y² = 1, z = 3.

23. Find the equation of the cone formed by rotating the line 2x + 3y = 6, z = 0 about the y-axis.

24. Find the equation to the sphere through the points (0, 0, 0), (0, 1, –1), (–1, 2, 0) and (1, 2, 3).

25. If a >0 be a real number then prove that $\frac{1}{a}$> 0.

26. Find the domain of f(g(x)) when f(x) = $\frac{1}{x+2}$ and g(x) = $\frac{1}{x+3}$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Define equivalence relation. Let R be the relation on the set of real numbers such that aRb iff a – b is an integer. Is R an equivalence relation ? Justify.

28. Prove that the equation $\sqrt{fx} \pm \sqrt{gy} \pm \sqrt{hz} = 0$ represents a cone that touches the co-ordinate planes; and that the equation to the reciprocal cone is fyz + gzx + hxy = 0

29. Prove that the plane x + 2y – z = 0 cuts the sphere x² + y² + z² – x + z – 2 = 0 in a circle of radius unity and find the equation of sphere which has this circle for one of its great circle.

30. Let A be a set. Prove that "There is no injective map f : P(A) → A and there is no surjective map g : A → P(A)".

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