Reg. No. : ....

Name : ....

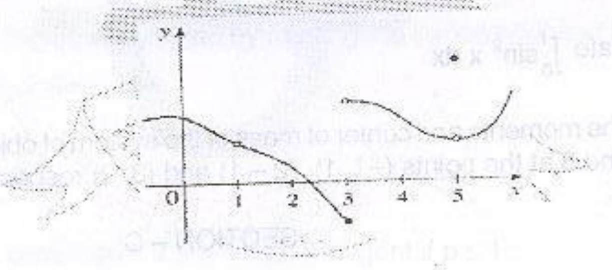
I Semester B.Sc. Honours in Mathematics (CBCSS – OBE – Regular) Examination, November 2022
(2021 Admission Onwards)
Core Course
1B01 BMH : CALCULUS – 1

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of **five** questions. **Each** question carries **one** mark.

1. Who gave the notion of e to exponential function ?
2. Figure below shows the graph of a function f. At which numbers is f discontinuous ?



3. Find $\frac{d}{dx}(\tanh^{-1}(\sin x))$.
4. State Fundamental theorem of calculus.
5. Set up, but do not evaluate, an integral for the length of the curve, $y = \cos x$, $0 \leq x \leq 2\pi$.

P.T.O.

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. Find the domain and range of $\frac{1}{2}e^{-x} - 1$.
7. Estimate the value of $\lim_{t \to 0} \frac{\sqrt{t^2+9} - 3}{t^2}$.
8. The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of atmost 0.05 cm. What is the maximum error in using this value of the radius to compute the volume of the sphere ?
9. Prove the identity $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$.
10. Calculate $\lim_{x \to \infty} \frac{e^x}{x^2}$.
11. Set up an expression for $\int_1^3 e^x \, dx$ as a limit of sums.
12. Evaluate $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$.
13. Evaluate $\int_0^1 \sin^2 x \, dx$.
14. Find the moments and center of mass of the system of objects that have masses 3, 4 and 8 at the points (–1, 1), (2 – 1) and (3, 2) respectively.

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. Suppose that a ball is dropped from the upper observation deck of the CN Tower in Toronto, 450 m above the ground. Find the velocity of the ball after 5 seconds.
16. Use ϵ, δ definition to prove $\lim_{x \to 3} x^2 = 9$.
17. Where is the function $f(x) = \frac{(\ln x + \tan^{-1} x)}{x^2 - 1}$ continuous ?

18. If f and g are continuous at a, then show that f + g is continuous at a.
19. Show that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$.
20. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 3x^2 + 1, \ -\frac{1}{2} \leq x \leq 4$.
21. Verify that the function $f(x) = x^3 + x - 1$, satisfies the hypotheses of the Mean Value Theorem on the interval [0, 2]. Then find all numbers that satisfy the conclusion of the Mean Value Theorem.
22. Let $f(x) = \frac{2x^2}{x^2 - 1}$.
    i) Find domain of f.
    ii) Find vertical and horizontal asymptote.
23. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.
24. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.
25. Calculate $\int_0^1 \tan^{-1} x \, dx$.
26. Show that $\int_1^\infty \frac{1}{x^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. Find the horizontal and vertical asymptotes of the graph of the function $f(x) = \frac{\sqrt{2x^2+1}}{3x-5}$.

28. Discuss the curve $y = x^4 - 4x^3$ with respect to concavity, points of inflection and local maxima and minima.
29. A tank has the shape of an inverted circular cone with height 10 m and base radius 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m³).
30. i) The curve $y = \sqrt{4 - x^2}, -1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x-axis.
    ii) Find the centroid of the region bounded by the curves $y = \cos x$, $y = 0$, $x = 0$ and $x = \frac{\pi}{2}$.