I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – Supplementary/Improvement) Examination, November 2022

(2016 – 2020 Admissions)

BHM 104 : MATRICES AND PROBABILITY THEORY

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any 4** questions. **Each** question carries **1** mark.

1. Find the length of the vector a = [4, –3, 0, 2].

2. State Cauchy Schwartz inequality.

3. Define eigenvalue of an n × n matrix A.

4. Define random variable.

5. When do we say that a distribution is symmetric ?   (4×1=4)

SECTION – B

Answer **any 6** questions out of **9** questions. **Each** question carries **2** marks.

6. Find the angle between the vectors x = [6, –4] and y = [–2, 3].

7. Find the reduced row echelon form of the matrix $\begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}$.

8. Find the inverse of the matrix $\begin{bmatrix} -5 & 2 \\ 9 & -4 \end{bmatrix}$.

9. Evaluate the determinant $\begin{bmatrix} 4 & 2 & 0 & 1 \\ 0 & 3 & 9 & 6 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 0 & 7 \end{bmatrix}$.

10. Find the characteristic polynomial of the matrix $\begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}$.

11. Examine whether $f(x) = \begin{cases} 2x, 0 < x \le 1 \\ 4 - 2x, 1 < x < 2 \\ 0, \text{elsewhere} \end{cases}$ is a density function.

12. A continuous random variable X has density function
$f(x) = \begin{cases} ax, 0 \le x \le 1 \\ a, 1 \le x \le 2 \\ -ax + 3a, 2 \le x \le 3 \\ 0, \text{elsewhere} \end{cases}$, determine the constant a.

13. Suppose that the probability that sets consisting of 1,2,3,4 and 5 persons pay a visit to an art gallery are 0.2, 0.5, 0.2, 0.07 and 0.03 respectively. What is the expected number of persons per set ?

14. The first and second moments of a distribution about origin are 2 and 8 respectively. Find the variance.   (6×2=12)

SECTION – C

Answer **any 8** questions out of **12** questions. **Each** question carries **4** marks.

15. When do we say that a matrix is in reduced row echelon form ?

16. Find the inverse of the matrix $\begin{bmatrix} 2 & -6 & 5 \\ -4 & 12 & -9 \\ 2 & -9 & 8 \end{bmatrix}$.

17. Determine whether [5,17,–20] is in the row space of $\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$.

18. Is the matrix $B = \begin{bmatrix} 7 & 1 & -1 \\ -11 & -3 & 2 \\ 18 & 2 & -4 \end{bmatrix}$ diagonalizable ? Justify.

19. Prove that the determinant of an upper triangular matrix is the product of its main diagonal entries.

20. If $A = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$, find A^{-3}.

21. A continuous random variable X has pdf f(x) = 2x, 0 < x ≤ 1 and 0 elsewhere. Find F(x), $P\left(X \le \frac{1}{2}\right)$, $P\left(\frac{1}{4} \le X \le \frac{3}{4}\right)$.

22. For the random variable X with pdf given by $f(x) = \frac{3}{4}x(2-x), 0 \le x \le 2$, and 0 elsewhere, find the mean and variance.

23. The joint pdf of a two dimensional random variable (X,Y) is given by $f(xy) = \begin{cases} 2, 0 < x < y < 1 \\ 0, \text{elsewhere} \end{cases}$. Find the marginal and conditional density functions of X and Y.

24. Find the first four moments about the mean of the numbers 2,3,7,8 and 10.

25. Find the moment generating functions of
    a) f(x) = ae^{-ax}, a > 0, x > 0
    b) $f(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$.

26. The first three moments of a distribution about the origin are 1, 7 and 38 respectively. Obtain the coefficient of skewness.   (8×4=32)

SECTION – D

Answer **any 2** questions out of **4** questions. **Each** question carries **6** marks.

27. Using Gaussian elimination method solve the system of linear equations
    5x – 5y – 15z = 40, 4x – 2y – 6z =19, 3x – 6y – 17z = 41.

28. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$.

29. For the joint distribution $f(xy) = \frac{2}{3}(1+x)e^{-y}, 0 < x < 1, y > 0$, find the conditional distribution x given y =1 and that of y given $x = \frac{1}{3}$.

30. State and prove Chebyshev's inequality.   (2×6=12)