K22U 3445

Reg. No. : ....

Name : ....

I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. – O.B.E. – Regular) Examination, November 2022
(2021 Admission Onwards)
Core Course
1B03BMH : LOGIC, SETS AND PROBABILITY THEORY

Time : 3 Hours   Max. Marks : 60

SECTION – A

Answer **any four** questions from the following. **Each** question carries **1** mark.

1. What is a random variable ?
2. Find E(X) if $f(x) = 30x^4(1 - x)$ when $0 \leq x \leq 1$ and 0 elsewhere.
3. Show that $M_{cX}(t) = M_X(ct)$, c being a constant.
4. Does the sentence "2+3=7" is a mathematical sentence ? Justify your answer.
5. Write the sentence "The square of an odd integer is odd" in symbols.

SECTION – B

Answer **any six** questions. **Each** question carries **2** marks.

6. A continuous random variable X has a p.d.f. $f(x) = kx^2, 0 \leq x \leq 1$. Find k.
7. If X is a random variable and a and b are constants. Prove that $E(aX + b) = aE(X) + b$ provided all the expectations exist.
8. Find the expectation of the number on a die when thrown.
9. If X and Y are independent random variables with mean 0 and –5 and variance 4 and 6 respectively. Find a and b such that aX + bY will have mean 0 an variance 28.
10. Let X be a random variable with p.d.f. f(x) = 2/3 when x = 1, 1/3 when x = 2 and 0 elsewhere. Find the moment generating function.
11. Define cumulants and obtain the first two cumulants in terms of central moments.
12. What is an implication ? Give an example.

P.T.O.

13. Give proof for the following sentence by using the contrapositive method. The square of an integer is odd, then the integer itself is odd.
14. Given that $A = \{(x, y) \in R^2 : 2x + y = 0\}$ and $B = \{(x, y) \in R^2 : y^2 = x\}$. Find $A \cap B$.

SECTION – C

Answer **any eight** questions. **Each** question carries **4** marks **each**.

15. From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Answer the following when the sample is drawn without replacement.
    a) Find the probability distribution of X.
    b) Find $P(X > 1)$ and $P(0 < X < 2)$.
16. Let X be a continuous, random variate with p.d.f. $f(x) = \begin{cases} ax, 0 \leq x \leq 1 \\ a, 1 \leq x \leq 2 \\ -ax + 3a, 2 \leq x \leq 3 \end{cases}$ and 0 otherwise.
    i) Determine the constant a.
    ii) Compute $P(X \leq 1.5)$.
17. A random variable X has the following probability distribution :

x	p(X)
0	0
1	k
2	2k
3	2k
4	3k
5	k^2
6	$2k^2$
7	$7k^2 + k$

determine the distribution function of X.

18. A coin is tossed until a head appears. What is the expectation of the number of tosses required ?
19. Let variate X have the distribution $P(X = 0) = P(X = 2) = p$ and $P(X = 1) = 1 - 2p, 0 \leq x \leq 1/2$. For what p is the Var(X) a maximum ?