Reg. No.:....

Name:

I Semester B.Sc. Honours in Mathematics (C.B.C.S.S. - Supplementary/ Improvement) Examination, November 2022 (2016-2020 Admissions)

BHM 105: TWO DIMENSIONAL GEOMETRY

Time: 3 Hours

Max. Marks: 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark:

- 1. Transform to parallel axes through the point (3, 5) the equation $x^2 + y^2 - 6x - 10y - 2 = 0$.
- 2. Find the distance between the points (5, 1) and (6, 0).
- Define parabola.
- Define conjugate hyperbola.
- Define auxiliary circle of an ellipse.

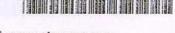
SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 marks :

- Show that the two lines represented by $x^{2}(\tan^{2}\theta + \cos^{2}\theta) - 2xy \tan\theta + y^{2}\sin^{2}\theta = 0$ make angles α , β with x axis such that $\tan \alpha - \tan \beta = 2$. 7. Find the value of λ so that the equation $2x^2 + xy - y^2 - 11x - 5y + \lambda = 0$ may
- represent a pair of lines. 8. Find the equation of the parabola with vertex (-1, -2), axis parallel to y-axis
- and passing through (3, 6).

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- Find the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2 = 4ax$.
- Prove that if tangents be drawn to the parabola y² = 4ax from a point on the line x + 4a = 0, their chord of contact will subtend a right angle at the vertex.
- 11. Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, focus is (-1, 1), directrix is x - y + 3 = 0. 12. Identify the conic $2x^2 - 3xy + 5y^2 + 6x - 3y + 5 = 0$.
- 13. Find the centre of the conic $14x^2 4xy + 11y^2 44x 58y + 71 = 0$.
- 14. Find the equation of an asymptote of the hyperbola $\frac{x^2}{a^2} \frac{y^2}{b^2} = 1$.
- SECTION C

Answer any 8 questions out of 12 questions. Each question carries 4 marks:

15. Find the equation of the pair of lines through the origin which represents the

- lines perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$. 16. Find the equation of the pair of lines bisecting the angles between the lines $ax^2 + 2hxy + by^2 = 0.$
- 17. If the chord joining 2 points whose eccentric angles are α and β on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cuts the major axis at a distance d from the centre. Show that $\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{d-a}{d+a}$.

respectively. Prove that $\frac{1}{e^2} + \frac{1}{e^{2}} = 1$.

18. Prove that product of perpendiculars from any point of the hyperbola $\frac{\chi^2}{a^2} - \frac{y^2}{b^2} = 1$ to the two asymptotes is equal to $\frac{a^2b^2}{a^2+b^2}$.

Find the vertex, focus, directrix and length of latus rectum of the parabola $5x^2 + 24y = 0$.

If e, e' be the ecentricities of a hyperbola and of the conjugate hyperbola

23. Identify the conic and write the equation of the directrix whose polar equation

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is $r = \frac{1}{1 + \cos \theta}$. 24. P(x, y) is equidistant from the points A (2,3) and B(3, - 1). Find the equation of the locus of P.

21. Prove that the locus of the poles of tangents to the parabola $y^2 = 4ax$ with

respect to the circle $x^2 + y^2 - 2ax = 0$ is the circle $x^2 + y^2 - ax = 0$.

22. Derive the polar equation of a conic with focus at the pole.

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- 25. Determine the equation of the curve $4x^2 11xy + 6y^2 = 0$ when the axes are rotated through the acute angle whose tangent is $\frac{4}{3}$ 26. Find the general equation of a parabola whose focus is (α, β) and directrix is
 - ax + by + c = 0. SECTION - D

Answer any 2 questions out of 4 questions. Each question carries 6 marks :

- 27. Trace the conic $9x^2 + 24xy + 16y^2 2x + 14y + 1 = 0$.
- 28. The co-ordinate axes are to be rotated through an angle α to produce an equation for the curve $2x^2 + \sqrt{3}xy + y^2 - 10 = 0$ that has no cross product term. Find α and the new equation and identify the curve.
- 29. Find the equation of tangent to a parabola $y^2 = 4ax$ at a point on it.
- 30. Derive the equation of director circle of the ellipse $\frac{\chi^2}{a^2} + \frac{y^2}{b^2} = 1$.