K24P 1103

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C06/MSMAF02C06 : Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. State and prove fundamental theorem of arithmetic.
2. Prove that every Euclidean domain is a PID.
3. Define algebraic number. Prove that $\sqrt{1+\sqrt{3}}$ is an algebraic number.
4. Find the degree and basis for $\mathbb{Q}(\sqrt[3]{2}, \sqrt{3})$ over $\mathbb{Q}$.
5. Prove that regular 20-gon is constructible.
6. Find the number of primitive 15^{th} roots of unity in GF(31).

PART – B

Answer **three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. State and prove Gauss's lemma.
8. Let E be a simple extension $F(\alpha)$ of a field F and let α be algebraic over F. Let the degree of $irr(\alpha, F)$ be $n \geq 1$. Prove that every element β of $E = F(\alpha)$ can be uniquely expressed in the form $\beta = b_0 + b_1\alpha + ... + b_{n-1}\alpha^{n-1}$, where the b_i are in F.
9. Prove that $\mathbb{R}[x]/\langle x^2+1\rangle$ is isomorphic to $\mathbb{C}$.

P.T.O.

10. Find the splitting field of $\{x^2 - 2, x^2 - 3\}$ over $\mathbb{Q}$ and what is the order of $G(\mathbb{Q}(\sqrt{2},\sqrt{3})/\mathbb{Q})$?
11. Define perfect field. Is every field of characteristic zero perfect ? Justify.

PART – C

Answer **three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. a) Prove that an ideal $\langle p\rangle$ in a PID is maximal if and only if p is an irreducible.
    b) Prove that every PID is a UFD. What about its converse ? Justify.
13. Prove that $\mathbb{Z}[\sqrt{-5}] = \{a + ib\sqrt{5} : a, b \in \mathbb{Z}\}$ is an integral domain but not a UFD.
14. State and prove Kronecker's theorem.
15. a) Prove that a field E, where $F \leq E \leq \overline{F}$ is a splitting field over F if and only if every automorphism of $\overline{F}$ leaving F fixed maps E on to itself and thus induces an automorphism of E leaving F fixed.
    b) Show that for a prime p, the splitting field over $\mathbb{Q}$ of $x^p - 1$ is of degree $p - 1$ over $\mathbb{Q}$.
16. a) State and prove primitive element theorem.
    b) State main theorem of Galois theory.

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