K24P 3124

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree (CBCSS – OBE – Regular)
Examination, October 2024
(2023 Admission)

MATHEMATICS/MATHEMATICS (MULTIVARIATE CALCULUS AND MATHEMATICAL ANALYSIS, MODELLING AND SIMULATION, FINANCIAL RISK MANAGEMENT

MSMAT03C11/MSMAF03C11 : Functional Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from Part – **A. Each** question carries **4** marks.

1. Prove that the norm $x \mapsto \|x\|$ is continuous mapping of $(X, \|\cdot\|)$ into R.
2. Show that the space C[a, b] is a Banach space with norm $\|x\| = \max_{t \in J}|x(t)|$, where $J \in [a, b]$. Is C[a, b] complete ?
3. Can every metric on a vector space be obtained from a norm ? Justify your answer.
4. Prove that a compact subset M of a metric space is closed and bounded.
5. Define sesquilinear functional. Prove that the inner product is sesquilinear and bounded.
6. Define self-adjoint, unitary and normal operators. Is a normal operator be self adjoint or/and unitary ? Justify your answer. **(5×4=20)**

PART – B

Answer **any three** questions. **Each** question carries **7** marks.

7. Prove : Every finite dimensional subspace Y of a normed linear space X is complete. In particular, every finite dimensional normed space is complete.

P.T.O.

8. If a normed space X has the property that the closed unit ball $M = \{x / \|x\| \leq 1\}$ is compact then, prove that X is finite dimensional.
9. If a normed space X is finite dimensional then, prove that every linear operator on X is bounded.
10. Prove that the space l^p with $p \neq 0$ is not an inner product space. Is the space l^p with $p \neq 0$ a Hilbert space ? Justify your answer.
11. Prove that the dual space of l^1 is l^∞. **(3×7=21)**

PART – C

Answer **any three** questions. **Each** question carries **13** marks.

12. Prove : For any inner product space X, there exists a Hilbert space H and an isomorphism from A from X onto a dense subspace $W \subset H$. The space H is unique except for isomorphisms.
13. Let H over K. Then, prove that :
    i) If H separable then every orthonormal set in H is countable.
    ii) If H contains an orthogonal sequence which is total H then H is separable.
14. Prove that two Hilbert spaces H and $\tilde{H}$ both real or complex are isomorphic if and only if they have the same Hilbert dimension.
15. Prove : Let $T : H \to H$ be a bounded linear operator, on a Hilbert space H. Then,
    a) If T is self adjoint, $\langle Tx, x\rangle$ is real for all $x \in H$.
    b) If H is complex and $\langle Tx, x\rangle$ is real for all $x \in H$, the operator T is self-adjoint.
16. Prove that in every Hilbert space $H \neq \{0\}$, there exists a total orthogonal set. **(3×13=39)**

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